Problem E1- Solution

Heat Conduction in a Copper Rod (10 points)

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

Part A: The short copper rod (3.9 points)

A.1 (0.2 pt)

$R_{en} = 110.11 \pm 0.01\ \Omega$

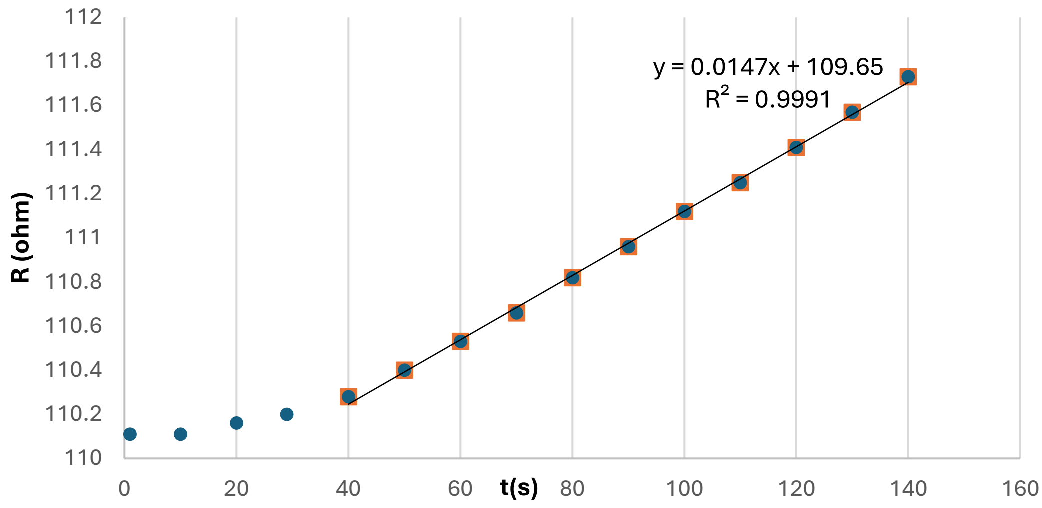
$$\theta_{en} = \frac{R - R_0}{R_0 \alpha}$$

$\theta_{en} = 25.87 \pm 0.03\ ℃$

A.2 (0.5 pt)

n	$R(\Omega)$	$t(s)$
1	110.11	1
2	110.11	10
3	110.15	20
4	110.18	30
5	110.26	40
6	110.4	50
7	110.53	60
8	110.66	70
9	110.82	80
10	110.96	90
11	111.12	100
12	111.25	110
13	111.41	120
14	111.57	130
15	111.73	140

A.3 (0.8 pt)

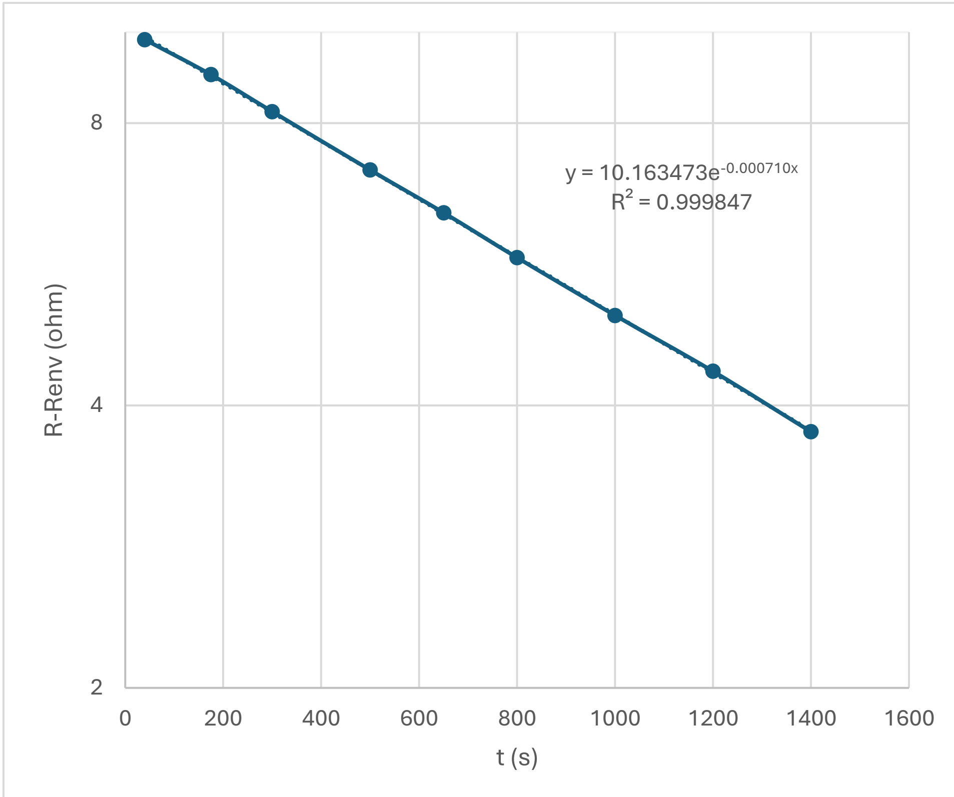


$\Delta R/\Delta t$	$0.0147\ \Omega/s$	$\frac{\Delta\theta}{\Delta t} = \frac{1}{R_0\alpha}\frac{\Delta R}{\Delta t}$
$\Delta\theta/\Delta t$	$0.03761\ ℃/s$	
C_s	$52 \pm 2\ \mathrm{J/℃}$	$C_s = \frac{P_1}{\frac{\Delta\theta}{\Delta t}}$, $\Delta C_s \approx C_s\frac{\Delta P_1}{P_1}$

A.4 (0.5pt)

n	$R(\Omega)$	$t(s)$	$(R-R_{en})(\Omega)$
1	119.98	40	9.82
2	119.17	175	9.01
3	118.39	300	8.23
4	117.29	500	7.13
5	116.58	650	6.42
6	115.91	800	5.75
7	115.15	1000	4.99
8	114.51	1200	4.35
9	113.91	1400	3.75
10	113.40	1600	3.24

A.5 (0.7 pt)

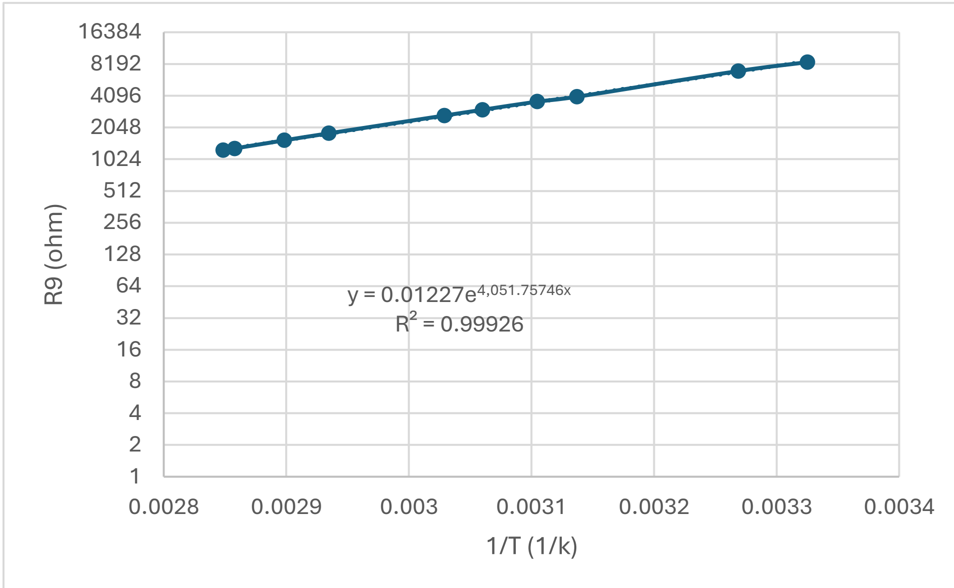


$\gamma = (710 \pm 3) \times 10^{-6}\ s^{-1}$

A.6 (0.5 pt)

n	$R(\Omega)$	$R_9(\Omega)$	$T(k)$	$\frac{1}{T}\left(\frac{1}{k}\right)$
1	8528	110.78	300.733	0.003325
2	7020	112.81	305.9272	0.003269
3	4005	117.83	318.772	0.003137
4	3601	119.13	322.0984	0.003105
5	3014	120.96	326.7808	0.00306
6	2658	122.27	330.1328	0.003029
7	1803	126.42	340.7515	0.002935
8	1547	128.09	345.0245	0.002898
9	1296	130.00	349.9117	0.002858
10	1246	130.45	351.0631	0.002848

A.7 (0.7 pt)



$E_g = 0.698 \pm 0.007\ eV$

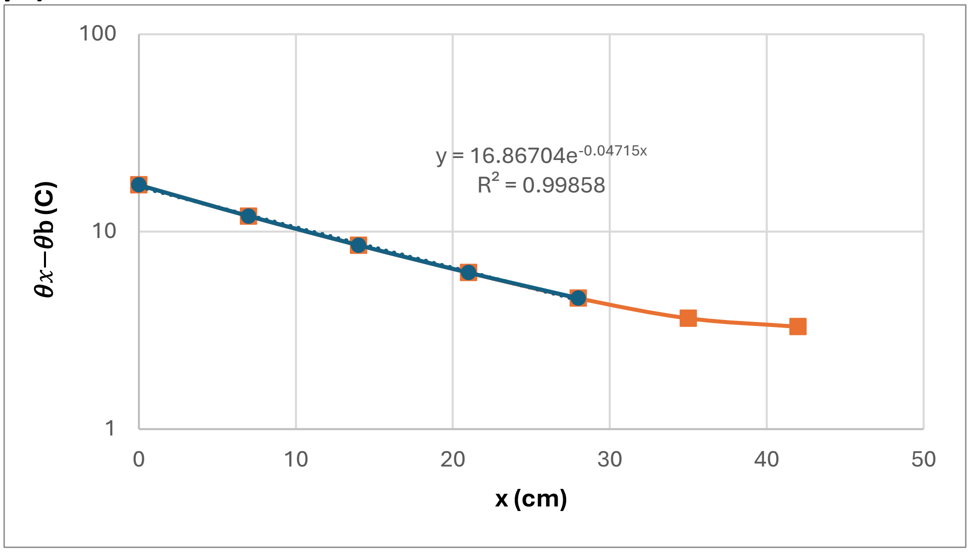
Part B: The long copper rod (4.1 points)

B.1 and B.5

$\theta_b = \ 24.41℃$

B.1 (0.4pt)				B.5 (0.4pt)	
n	$x(cm)$	$\theta_x(℃)$	$\theta_x - \theta_b(℃)$	$B^{(1)}e^{\lambda^{(0)}x}(℃)$	$\theta'_x - \theta_b(℃)$
1	0	44.61	17.23	0.27	16.96
2	7	39.37	11.99	0.37	11.62
3	14	35.92	8.54	0.51	8.03
4	21	33.58	6.20	0.72	5.48
5	28	31.98	4.60	1.00	3.60
6	35	31.02	3.64	1.39	2.25
7	42	30.68	3.30	1.93	1.37

B.2 (0.4 pt)



B.3 (0.6 pt)

$A^{(0)} = 16.87\ ℃$

$\lambda^{(0)} = 0.047 \pm\ 0.005 \quad \left(\frac{1}{cm}\right)$

B.4 (0.4 pt)

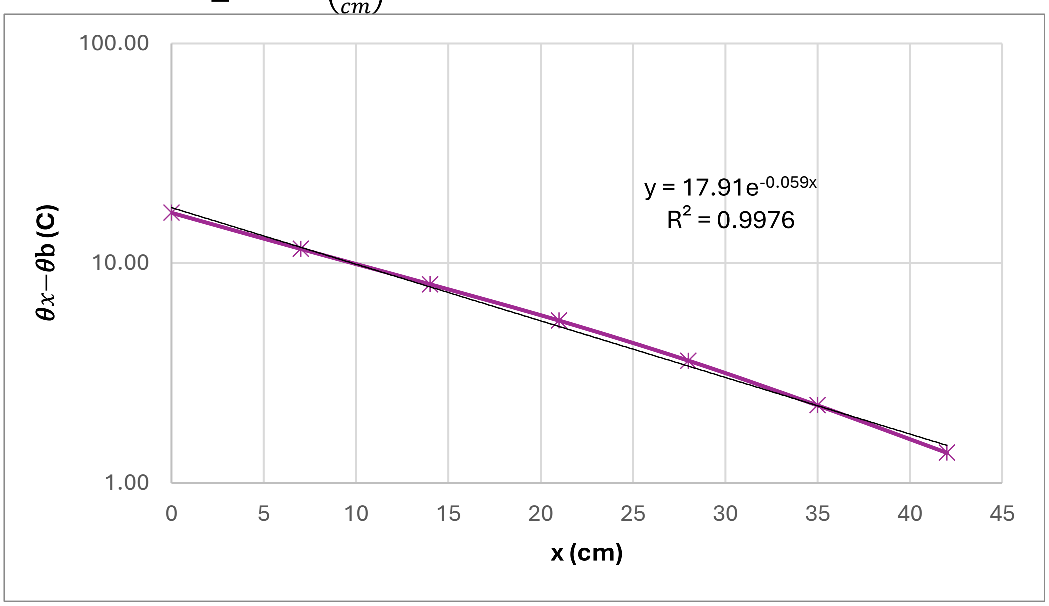
$B = Ae^{-2\lambda d}$

$B^{(1)} = 0.266℃$

B.6 (1.0 pt)

$A^{(1)} = 17.91℃$

$\lambda^{(1)} = 0.059 \pm\ 0.002 \left(\frac{1}{cm}\right)$



B.7 (0.9 pt)

$$P_2 = \int_{-0.5cm}^{42.5cm} 2\pi r h(\theta_x - \theta_b)dx \quad \text{or} \quad \int_0^d 2\pi r h(\theta_x - \theta_b)dx = \frac{2\pi r h A}{\lambda}(1 - e^{-2\lambda d})$$

$$h = \frac{P_2 \lambda}{2\pi r A(1 - e^{-2\lambda d})}, \; k = \frac{2h}{\lambda^2 r}, P_2 = 4.5\ W$$

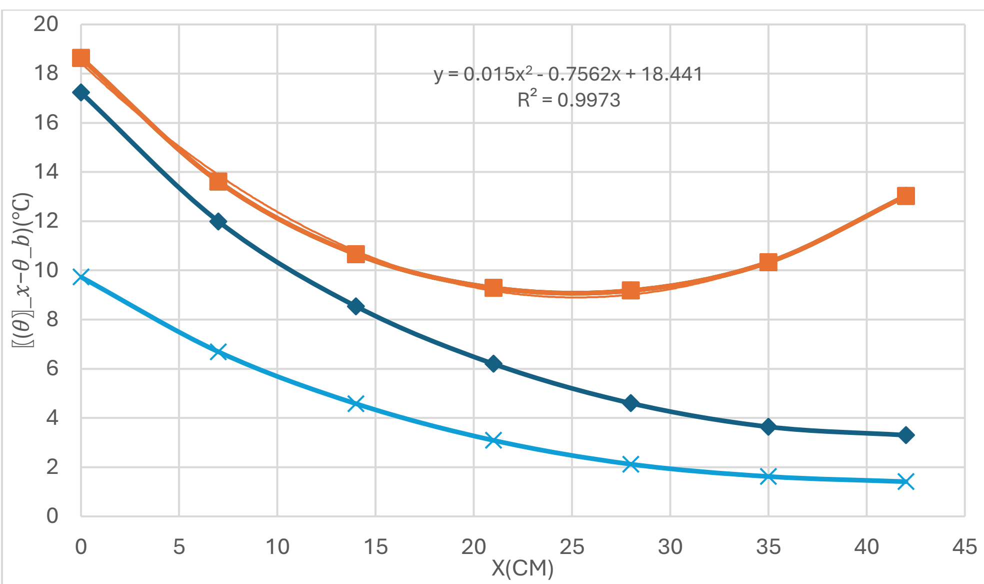
$$\lambda = 0.053 \pm 0.008 \left(\frac{1}{cm}\right), \; h = 0.0037 \pm 0.0003 \frac{W}{k\ cm^2}, k = 3.9 \pm 0.3 \frac{W}{k\ cm}$$

C.1 (0.4 pt)

$\theta_b = 27.38℃$

n	$x(cm)$	$\theta_x(℃)$	$(\theta_x - \theta_b)(℃)$	B-1	C-B	Reversed order
1	0	46.02	18.64	17.23	1.41	9.73
2	7	40.99	13.61	11.99	1.62	6.69
3	14	38.04	10.66	8.54	2.12	4.58
4	21	36.67	9.29	6.2	3.09	3.09
5	28	36.56	9.18	4.6	4.58	2.12
6	35	37.71	10.33	3.64	6.69	1.62
7	42	40.41	13.03	3.3	9.73	1.41

C.2 (0.6 pt)

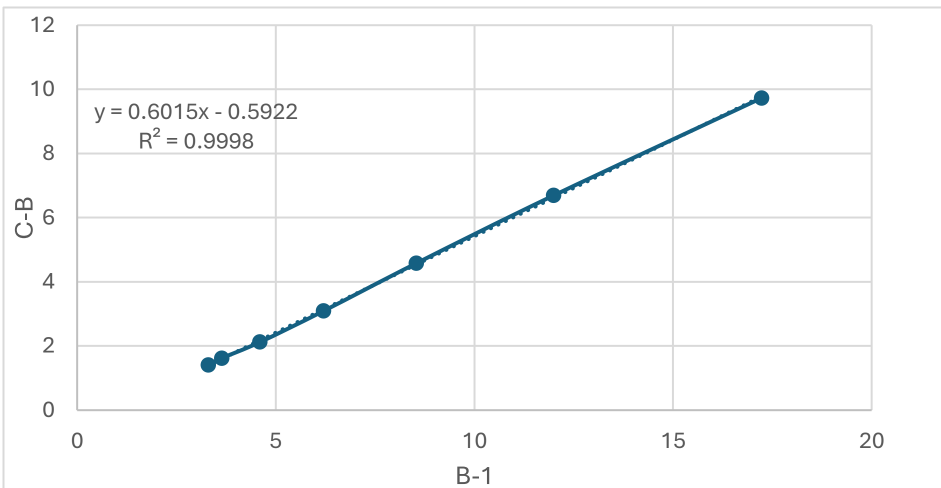


(The blue plots are not necessary to draw)

C.3 (1.0 pt)

1) $\frac{P_3}{P_2} = \frac{\frac{d\theta}{dx}_{x=42cm}}{\frac{d\theta}{dx}_{x=0cm}} = \frac{\frac{\Delta\theta}{\Delta x}_{x=42cm}}{\frac{\Delta\theta}{\Delta x}_{x=0cm}} = \frac{\theta_7 - \theta_6}{\theta_0 - \theta_1}$
2) $\frac{P_3}{P_2} = \frac{\frac{d\theta}{dx}_{x=42cm}}{\frac{d\theta}{dx}_{x=0cm}} = \frac{\sinh(\lambda(42-x_0))}{\sinh(\lambda x_0)}$
3) calculation by integral method or sigma in several parts

4) Slope ((C-B) _ B)



$$P_3 = 0.60\, P_2 \pm 0.01\, P_2$$