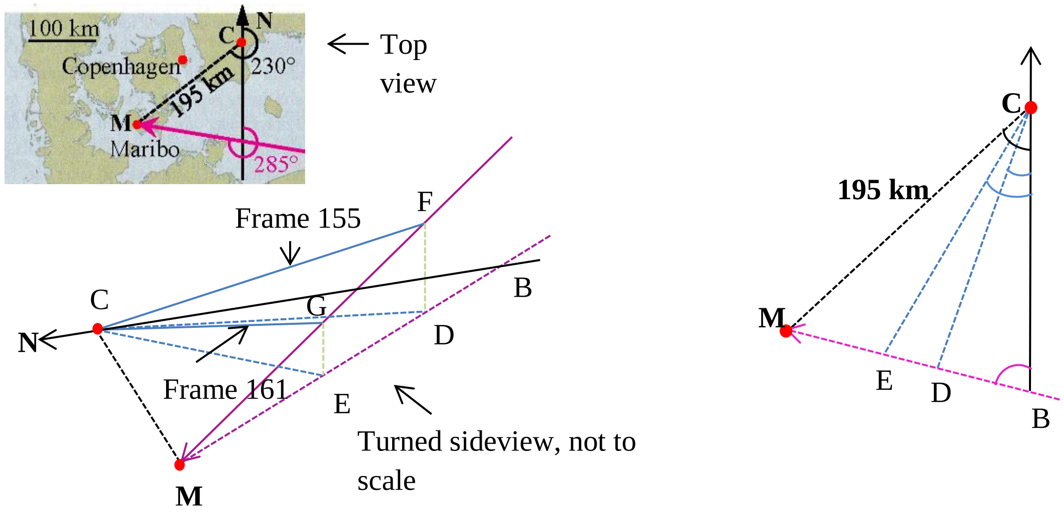
Solutions

1.1



Top view: Triangle MBC: $|\mathrm{CM}| = 195$ km, $\angle\mathrm{BCM} = 230° - 180° = 50°$, and $\angle\mathrm{MBC} = 75°$, so $\angle\mathrm{CMB} = 180° - 75° - 50° = 55°$.

Then $|\mathrm{CB}| = \frac{|\mathrm{CM}|\sin(\angle\mathrm{CMB})}{\sin(\angle\mathrm{MBC})} = 165.4$ km.

Triangle DBC: $|\mathrm{BC}| = 165.4$ km, $\angle\mathrm{BCD} = 215° - 180° = 35°$,and $\angle\mathrm{DBC} = 75°$, so $\angle\mathrm{CDB} = 180° - 75° - 35° = 70°$.

Then $|\mathrm{CD}| = \frac{|\mathrm{BC}|\sin(\angle\mathrm{DBC})}{\sin(\angle\mathrm{CDB})} = 170.0$ km.

Triangle EBC: $|\mathrm{BC}| = 165.4$ km, $\angle\mathrm{BCE} = 221° - 180° = 41°$, and $\angle\mathrm{EBC} = 75°$, so $\angle\mathrm{CEB} = 180° - 75° - 41° = 64°$.

Then $|\mathrm{CE}| = \frac{|\mathrm{BC}|\sin(\angle\mathrm{EBC})}{\sin(\angle\mathrm{CEB})} = 177.7$ km.

Triangle EDC: $\angle\mathrm{DCE} = 41° - 35° = 6°$. Horizontal distance traveled by Maribo: $|\mathrm{DE}| = \frac{|\mathrm{DC}|\sin(\angle\mathrm{DCE})}{\sin(\angle\mathrm{CED})} = 19.77$ km

Side view: Triangle CDF: $|\mathrm{DF}| = |\mathrm{DC}|\tan(\angle\mathrm{FCD}) = 59.20$ km

Triangle CEG: $|\mathrm{EG}| = |\mathrm{CE}|\tan(\angle\mathrm{GCE}) = 46.62$ km

Thus vertical distance travelled by Maribo: $|\mathrm{DF}| - |\mathrm{EG}| = 12.57$ km.

Total distance travelled by Maribo from frame 155 to 161:

$|\mathrm{FG}| = \sqrt{|\mathrm{DE}|^2 + (|\mathrm{DF}| - |\mathrm{EG}|)^2} = 23.43$ km.

The speed of Maribo is $v = \frac{23.43\ \mathrm{km}}{2.28\ \mathrm{s} - 1.46\ \mathrm{s}} = 28.6\ \mathrm{km/s}$

1.3

1.2a

Newton's second law: $m_\mathrm{M}\frac{\mathrm{d}v}{\mathrm{d}t} = -k\rho_\mathrm{atm}\pi R_\mathrm{M}^2 v^2$ yields $\frac{1}{v^2}\mathrm{d}v = -\frac{k\rho_\mathrm{atm}\pi R_\mathrm{M}^2}{m_\mathrm{M}}\,\mathrm{d}t$.

By integration $t = \frac{m_\mathrm{M}}{k\rho_\mathrm{atm}\pi R_\mathrm{M}^2}\left(\frac{1}{0.9} - 1\right)\frac{1}{v_\mathrm{M}} = 0.88$ s.

Alternative solution: The average force on the meteoroid when the speed decreases from v_M to $0.9\,v_\mathrm{M}$ can be estimated to $F_\mathrm{av} = -k\rho_\mathrm{atm}\pi R_\mathrm{M}^2\,(0.95\,v_\mathrm{M})^2$. Using that the acceleration is approximately constant, $a_\mathrm{av} = -k\rho_\mathrm{atm}\pi R_\mathrm{M}^2\,(0.95\,v_\mathrm{M})^2/m_\mathrm{M}$, results in $t = \frac{-0.1\,v_\mathrm{M}}{a_\mathrm{av}} = 0.87$ s.

0.7

1.2b	$\frac{E_{\mathrm{kin}}}{E_{\mathrm{melt}}} = \frac{\frac{1}{2}v_{\mathrm{M}}^2}{c_{\mathrm{sm}}(T_{\mathrm{sm}}-T_0)+L_{\mathrm{sm}}} = \frac{4.2\times10^8}{2.1\times10^6} = 2.1\times10^2 \gg 1.$	0.3

1.3a	$[x] = [t]^\alpha[\rho_{\mathrm{sm}}]^\beta[c_{\mathrm{sm}}]^\gamma\,[k_{\mathrm{sm}}]^\delta = [\mathrm{s}]^\alpha[\mathrm{kg\ m^{-3}}]^\beta[\mathrm{m^2\ s^{-2}K^{-1}}]^\gamma\,[\mathrm{kg\ m\ s^{-3}K^{-1}}]^\delta$, so $[\mathrm{m}] = [\mathrm{kg}]^{\beta+\delta}[\mathrm{m}]^{-3\beta+2\gamma+\delta}[\mathrm{s}]^{\alpha-2\gamma-3\delta}[\mathrm{K}]^{-\gamma-\delta}$. Thus $\beta+\delta=0,\ \ -3\beta+2\gamma+\delta=1,\ \ \alpha-2\gamma-3\delta=0$, and $-\gamma-\delta=0$. From which $(\alpha,\beta,\gamma,\delta) = \left(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2},+\frac{1}{2}\right)$ and $x(t)\approx\sqrt{\frac{k_{\mathrm{sm}}t}{\rho_{\mathrm{sm}}c_{\mathrm{sm}}}}$.	0.6
1.3b	$x(5\ \mathrm{s}) = 1.6\ \mathrm{mm}$ $\quad x/R_{\mathrm{M}} = 1.6\ \mathrm{mm}/130\ \mathrm{mm} = 0.012.$	0.4

1.4a	Rb-Sr decay scheme: ${}^{87}_{37}\mathrm{Rb} \to {}^{87}_{38}\mathrm{Sr} + {}^{\ 0}_{-1}\mathrm{e} + \bar{\nu}_{\mathrm{e}}$	0.3
1.4b	$N_{87\mathrm{Rb}}(t) = N_{87\mathrm{Rb}}(0)\mathrm{e}^{-\lambda t}$ and Rb→Sr: $N_{87\mathrm{Sr}}(t) = N_{87\mathrm{Sr}}(0) + [N_{87\mathrm{Rb}}(0) - N_{87\mathrm{Rb}}(t)]$. Thus $N_{87\mathrm{Sr}}(t) = N_{87\mathrm{Sr}}(0) + (\mathrm{e}^{\lambda t}-1)N_{87\mathrm{Rb}}(t)$, and dividing by $N_{86\mathrm{Sr}}$ we obtain the equation of a straight line: $\frac{N_{87\mathrm{Sr}}(t)}{N_{86\mathrm{Sr}}} = \frac{N_{87\mathrm{Sr}}(0)}{N_{86\mathrm{Sr}}} + (\mathrm{e}^{\lambda t}-1)\frac{N_{87\mathrm{Rb}}(t)}{N_{86\mathrm{Sr}}}$.	0.7
1.4c	Slope: $\mathrm{e}^{\lambda t} - 1 = a = \frac{0.712-0.700}{0.25} = 0.050$ and $T_{½} = \frac{\ln(2)}{\lambda} = 4.9\times10^{10}$ year. So $\tau_{\mathrm{M}} = \ln(1+a)\frac{1}{\lambda} = \frac{\ln(1+a)}{\ln(2)}T_{½} = 3.4\times10^9$ year .	0.4

1.5	Kepler's 3rd law on comet Encke and Earth, with the orbital semi-major axis of Encke given by $a = \frac{1}{2}(a_{\min}+a_{\max})$. Thus $t_{\mathrm{Encke}} = \left(\frac{a}{a_{\mathrm{E}}}\right)^{\frac{3}{2}} t_{\mathrm{E}} = 3.30\ \text{year} = 1.04\times10^8\ \mathrm{s}$.	0.6

1.6a	For Earth around its rotation axis: Angular velocity $\omega_{\mathrm{E}} = \frac{2\pi}{24\ \mathrm{h}} = 7.27\times10^{-5}\ \mathrm{s^{-1}}$. Moment of inertia $I_{\mathrm{E}} = 0.83\frac{2}{5}m_{\mathrm{E}}R_{\mathrm{E}}^2 = 8.07\times10^{37}\ \mathrm{kg\ m^2}$. Angular momentum $L_{\mathrm{E}} = I_{\mathrm{E}}\omega_{\mathrm{E}} = 5.87\times10^{33}\ \mathrm{kg\ m^2s^{-1}}$. Asteroid: $m_{\mathrm{ast}} = \frac{4\pi}{3}R_{\mathrm{ast}}^3\rho_{\mathrm{ast}} = 1.57\times10^{15}\ \mathrm{kg}$ and angular momentum $L_{\mathrm{ast}} = m_{\mathrm{ast}}v_{\mathrm{ast}}R_{\mathrm{E}} = 2.51\times10^{26}\ \mathrm{kg\ m^2s^{-1}}$. L_{ast} is perpendicular to L_{E}, so by conservation angular momentum: $\tan(\Delta\theta) = L_{\mathrm{ast}}/L_{\mathrm{E}} = 4.27\times10^{-8}$.The axis tilt $\Delta\theta = 4.27\times10^{-8}$ rad (so the North Pole moves $R_{\mathrm{E}}\,\Delta\theta = 0.27$ m).	0.7
1.6b	At vertical impact $\Delta L_{\mathrm{E}} = 0$ so $\Delta(I_{\mathrm{E}}\omega_{\mathrm{E}}) = 0$. Thus $\Delta\omega_{\mathrm{E}} = -\omega_{\mathrm{E}}(\Delta I_{\mathrm{E}})/I_{\mathrm{E}}$, and since $\Delta I_{\mathrm{E}}/I_{\mathrm{E}} = m_{\mathrm{ast}}R_{\mathrm{E}}^2/I_{\mathrm{E}} = 7.9\times10^{-10}$ we obtain $\Delta\omega_{\mathrm{E}} = -5.76\times10^{-14}\ \mathrm{s^{-1}}$. The change in rotation period is $\Delta T_{\mathrm{E}} = 2\pi\left(\frac{1}{\omega_{\mathrm{E}}+\Delta\omega_{\mathrm{E}}} - \frac{1}{\omega_{\mathrm{E}}}\right) \approx -2\pi\frac{\Delta\omega_{\mathrm{E}}}{\omega_{\mathrm{E}}^2} = 6.84\times10^{-5}\ \mathrm{s}$.	0.7
1.6c	At tangential impact L_{ast} is parallel to L_{E} so $L_{\mathrm{E}} + L_{\mathrm{ast}} = (I_{\mathrm{E}}+\Delta I_{\mathrm{E}})(\omega_{\mathrm{E}}+\Delta\omega_{\mathrm{E}})$ and thus $\Delta T_{\mathrm{E}} = 2\pi\left(\frac{1}{\omega_{\mathrm{E}}+\Delta\omega_{\mathrm{E}}} - \frac{1}{\omega_{\mathrm{E}}}\right) = 2\pi\left(\frac{I_{\mathrm{E}}+\Delta I_{\mathrm{E}}}{L_{\mathrm{E}}+L_{\mathrm{ast}}} - \frac{1}{\omega_{\mathrm{E}}}\right) = -3.62\times10^{-3}\ \mathrm{s}$.	0.7

1.7	Maximum impact speed $v_{\mathrm{imp}}^{\mathrm{max}}$ arises from three contributions: (I) The velocity v_{b} of the body at distance a_{E} (Earth orbit radius) from the Sun, $v_{\mathrm{b}} = \sqrt{\frac{2Gm_S}{a_E}} = 42.1\ \mathrm{km/s}$. (II) The orbital velocity of the Earth, $v_{\mathrm{E}} = \frac{2\pi a_E}{1\ \mathrm{year}} = 29.8\ \mathrm{km/s}$. (III) Gravitational attraction from the Earth and kinetic energy seen from the Earth: $\frac{1}{2}(v_{\mathrm{b}} + v_{\mathrm{E}})^2 = -\frac{Gm_E}{R_E} + \frac{1}{2}\left(v_{\mathrm{imp}}^{\mathrm{max}}\right)^2$. In conclusion: $v_{\mathrm{imp}}^{\mathrm{max}} = \sqrt{(v_{\mathrm{b}} + v_{\mathrm{E}})^2 + \frac{2Gm_E}{R_E}} = 72.8\ \mathrm{km/s}$.	1.6

	Total	**9.0**