Part 1. Calibration

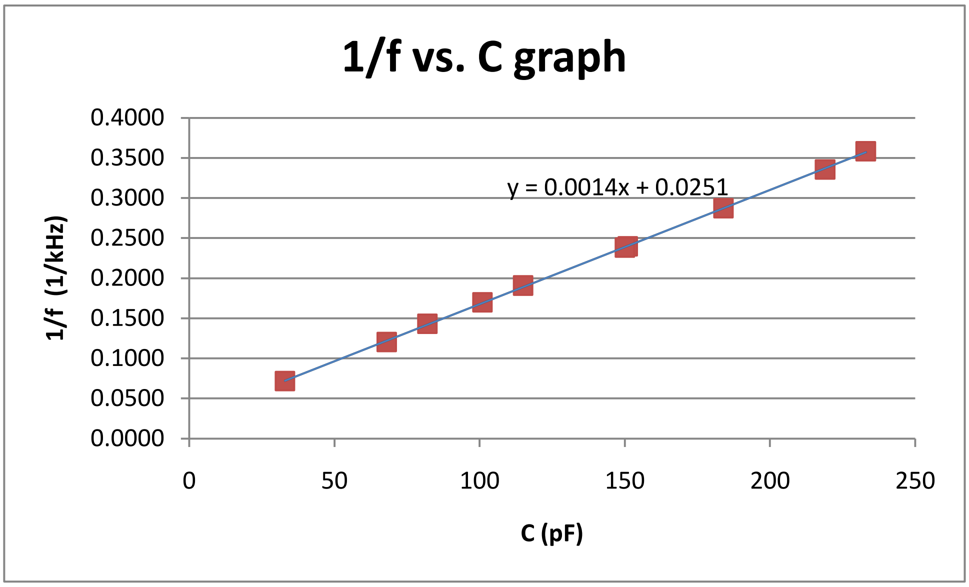
From the relationship between f and C given,

$$f = \frac{\alpha}{C + C_S} \qquad \Leftrightarrow \qquad \frac{1}{f} = \frac{1}{\alpha}C + \frac{C_S}{\alpha}$$

That is, theoretically, the graph of $\frac{1}{f}$ on the Y-axis versus C on the X-axis should be linear of which the slope and the Y-intercept is $\frac{1}{\alpha}$ and $\frac{C_S}{\alpha}$ respectively.

The table below shows the measured values of C (plotted on the X-axis,) f and, additionally, $\frac{1}{f}$, which is plotted on the Y-axis.

C (pF)	f (kHz)	1/f (ms)
33	13.94	0.0717
68	8.30	0.1205
82	6.99	0.1431
151	4.17	0.2398
233	2.79	0.3584
219	2.98	0.3356
184	3.48	0.2874
150	4.20	0.2381
115	5.24	0.1908
101	5.89	0.1698



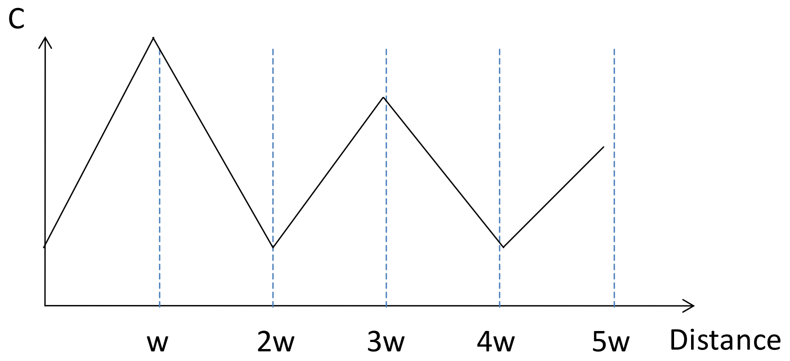
From this graph, the slope ($\frac{1}{\alpha}$) and the Y-intercept ($\frac{C_S}{\alpha}$) is equal to 0.0014 s/nF and 0.0251 ms respectively.

Hence, $\alpha = \frac{1}{\text{slope}} = \frac{1}{0.0014 \text{ s / nF}} = 714$ nF/s

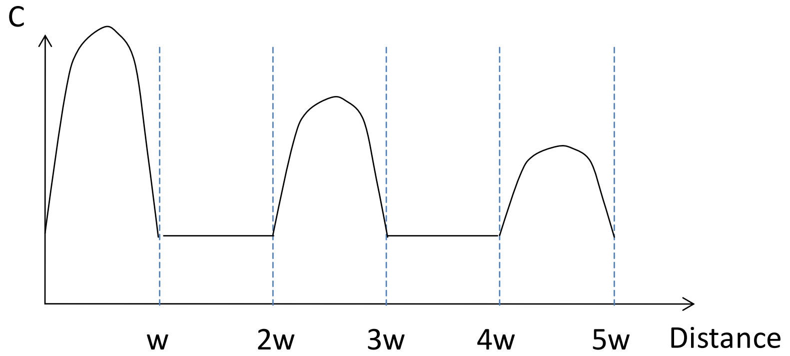
and $C_S = \frac{\text{Y} - \text{intercept}}{\text{slope}} = \frac{0.0251 \text{ ms}}{0.0014 \text{ s / nF}} = 17.9$ pF as required.

Part II. Determination of geometrical shape of parallel-plates capacitor

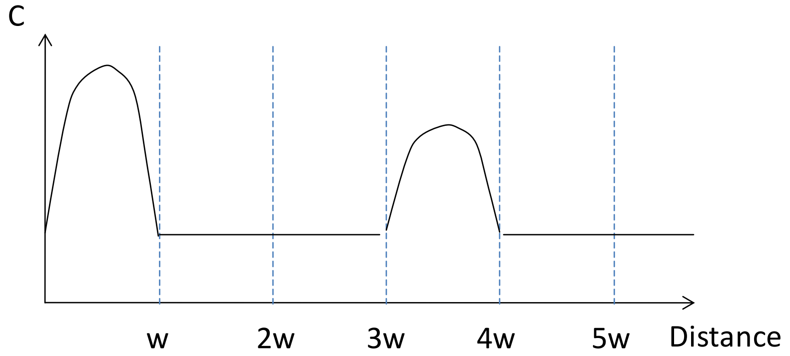
PATTERN I: The expected graph of C versus the position



PATTERN II: The expected graph of C versus the position

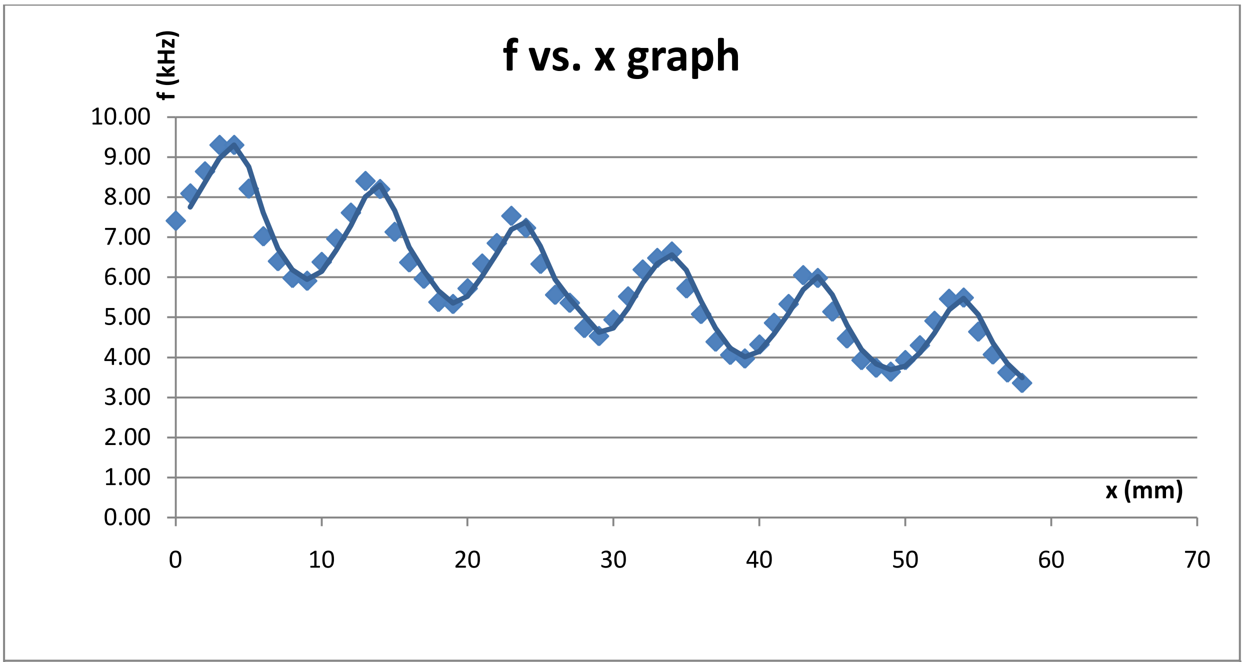


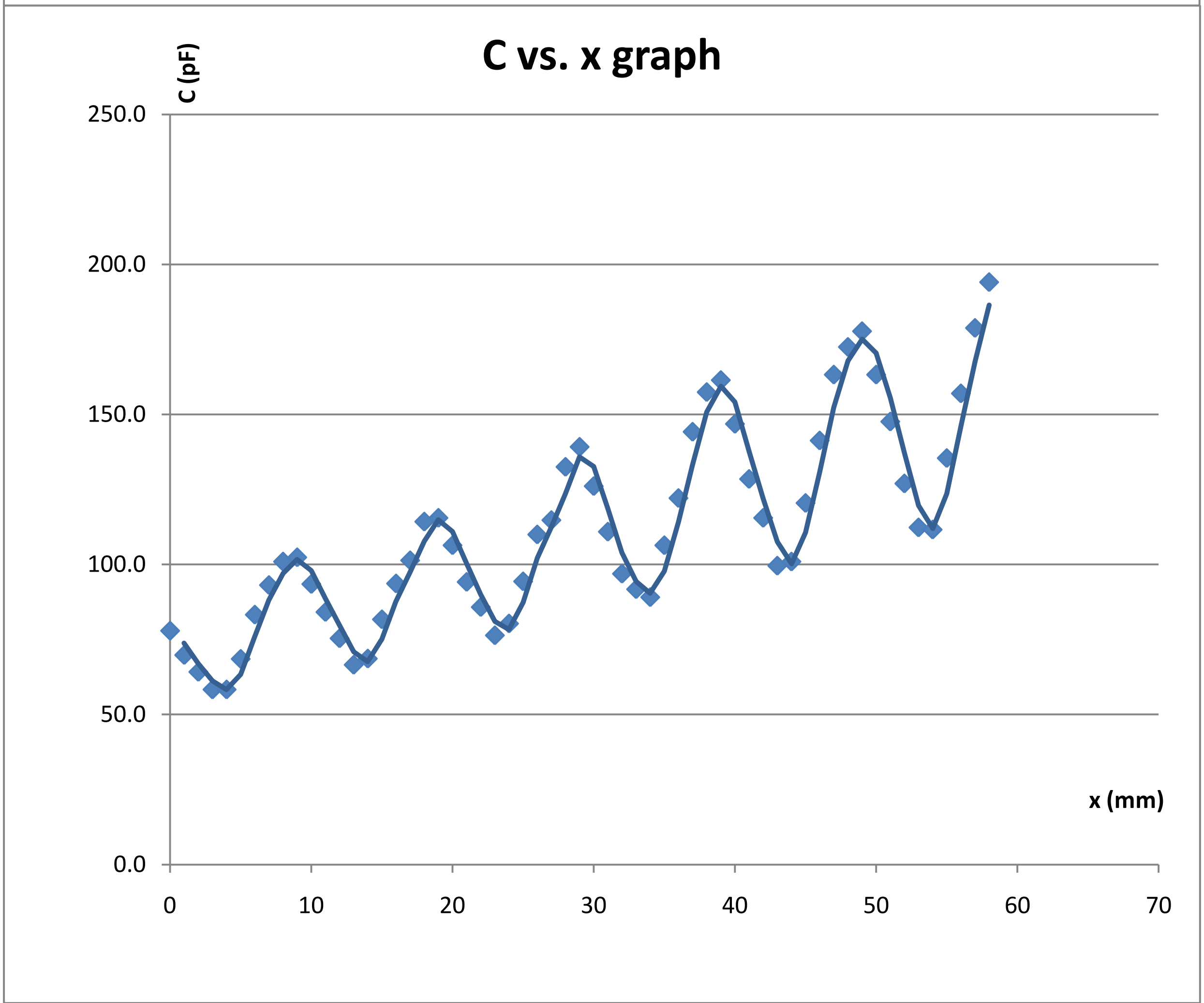
PATTERN III: The expected graph of C versus the position



By measuring f and C versus x (the distance moved between the two plates,) the data and the graphs are shown below.

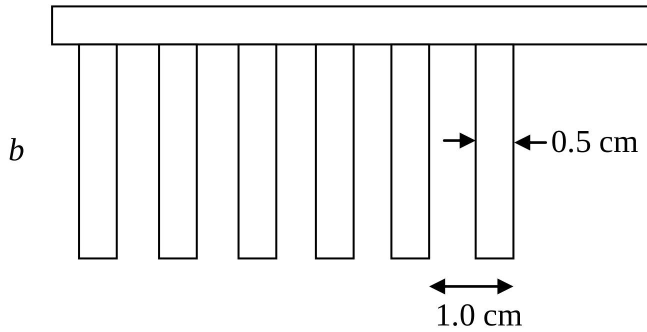
x (mm)	f (kHz)	C (pF)	x (mm)	f (kHz)	C (pF)
0	7.41	77.9	30	4.94	126.1
1	8.09	69.8	31	5.52	110.9
2	8.64	64.2	32	6.19	96.9
3	9.30	58.3	33	6.48	91.7
4	9.30	58.3	34	6.64	89.1
5	8.21	68.5	35	5.72	106.4
6	7.02	83.3	36	5.08	122.1
7	6.40	93.1	37	4.39	144.2
8	5.98	100.9	38	4.06	157.4
9	5.91	102.4	39	3.97	161.4
10	6.38	93.5	40	4.32	146.8
11	6.96	84.1	41	4.86	128.5
12	7.61	75.4	42	5.33	115.5
13	8.40	66.5	43	6.05	99.6
14	8.20	68.6	44	5.98	100.9
15	7.13	81.7	45	5.14	120.5
16	6.37	93.6	46	4.47	141.3
17	5.96	101.3	47	3.93	163.3
18	5.38	114.3	48	3.74	172.5
19	5.33	115.5	49	3.64	177.7
20	5.72	106.4	50	3.93	163.3
21	6.34	94.2	51	4.30	147.6
22	6.85	85.8	52	4.91	127.0
23	7.53	76.4	53	5.46	112.3
24	7.23	80.3	54	5.49	111.6
25	6.33	94.3	55	4.64	135.4
26	5.56	110.0	56	4.07	157.0
27	5.36	114.8	57	3.62	178.8
28	4.73	132.5	58	3.36	194.1
29	4.53	139.2			


f vs. x graph
f (kHz)
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
x (mm)
0
10
20
30
40
50
60
70


C vs. x graph
C (pF)
250.0
200.0
150.0
100.0
50.0
0.0
x (mm)
0
10
20
30
40
50
60
70

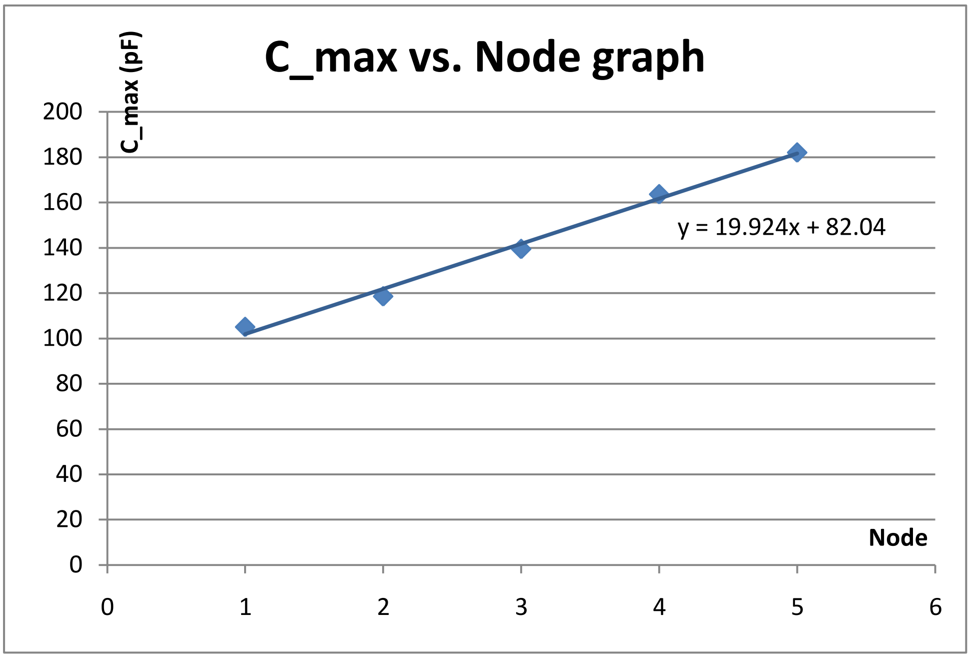
From periodicity of the graph, period = 1.0 cm

Simple possible configuration is:



The peaks of C values obtained from the C vs. x graph are provided in the table below. These maximum C are plotted (on the Y-axis) vs. nodes (on the X-axis.)

node	C_max
1	105.1
2	118.6
3	139.5
4	163.7
5	182.1



This graph is linear of which the slope is the dropped off capacitance $\Delta C = 19.9$ pF/section.

Given that the distance between the plates $d = 0.20$ mm, $K = 1.5$,

$$\Delta C \approx \frac{K\varepsilon_0 A}{d},$$

and $A = 5\times10^{-3}\,\mathrm{m} \times b\,\mathrm{mm} \times10^{-3}\ \mathrm{m}^2$

Then, $b \text{ mm} \approx \dfrac{\Delta C \ d}{K\varepsilon_0 \times 10^{-3} \times 5 \times 10^{-3}} \approx 60 \text{ mm}$ if medium between plates is the dielectric of which $K = 1.5$.

Part III. Resolution of digital micrometer

From the given relationship between f and C, $f = \dfrac{\alpha}{C + C_S}$,

$$\Delta f \simeq \left|\frac{df}{dC}\right| \Delta C = \left|\frac{-\alpha}{(C + C_S)^2}\right| \Delta C$$

$$= \frac{f^2}{\alpha} \Delta C$$

$$\Leftrightarrow \qquad \Delta C = \frac{\alpha}{f^2} \Delta f$$

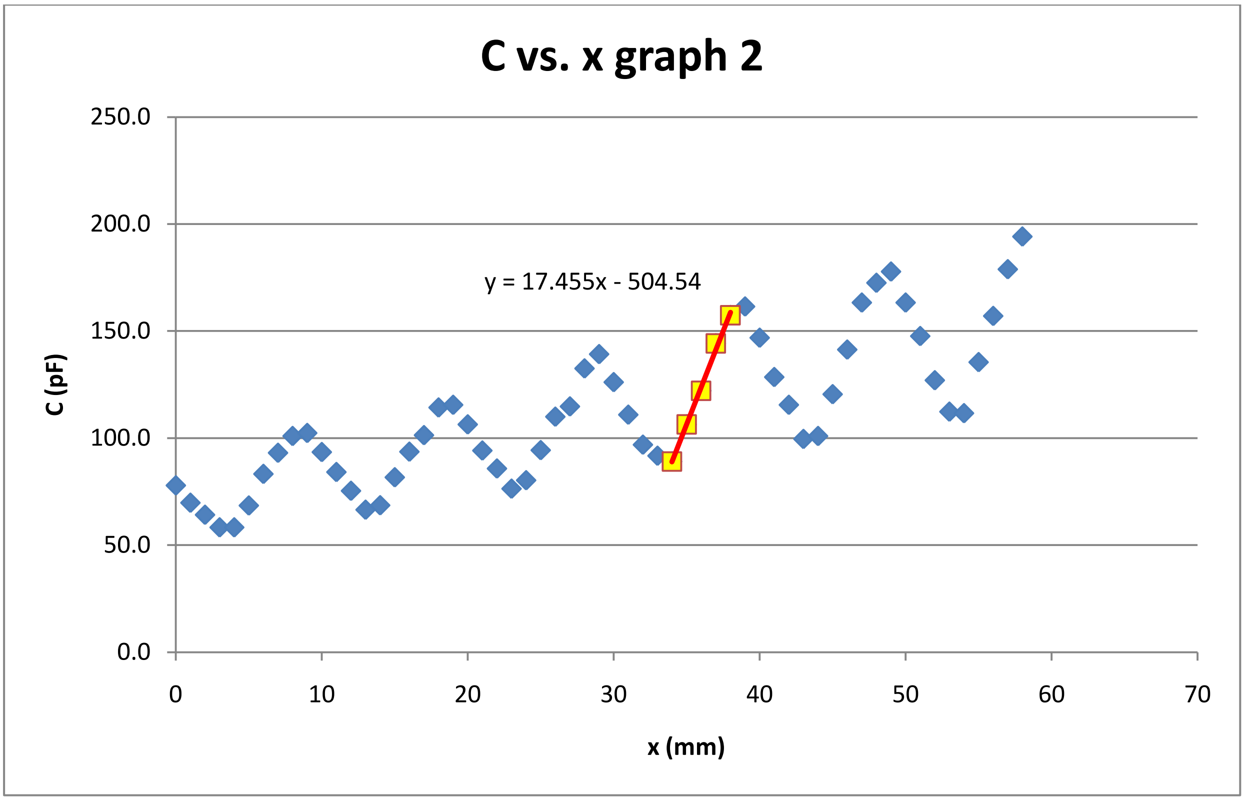
And since C linearly depends on x, $C = mx + \beta \quad \Rightarrow \quad \Delta C = m\Delta x$.

Hence,

$$\Delta x = \frac{\alpha}{mf^2} \Delta f,$$

where Δf is the smallest change of the frequency f which can be detected by the multimeter, x_0 is the operated distance at f = 5 kHz, and m is the gradient of the C vs. x graph at $x = x_0$.

From the f vs. x graph, at f = 5 kHz, The gradient is then measured on the C vs. x graph around this range.



From this graph, $m = 17.5\ \text{pF}/\text{mm} = 1.75\times10^{-8}\text{F}/\text{m}$.
Using this value of m, $f = 5\ \text{kHz}, \alpha = 714\ \text{nF/s}$, and $\Delta f = 0.01\ \text{kHz}$,

$$\Delta x = \frac{714\times10^{-9}}{(1.75\times10^{-8})(5\times10^{3})^{2}}\times(0.01\times10^{3}) = 0.016\,\text{mm}$$

NB. The C vs. x graph is used since C (but not f) is linearly related to x.

Alternative method for finding the resolution
(not strictly correct)

Using the f vs. x graph and the data in the table around $f = 5\ \text{kHz}$, it is found that when f is changed by 1 kHz ($\Delta f = 1\ \text{kHz}$,) x is roughly changed by 1.5 mm ($\Delta x \simeq 1.5\ \text{mm}$.) Hence, when f is changed by $\Delta f = 0.01\ \text{kHz}$ (the smallest detectable of the change,) the distance moved is $\Delta x \simeq 0.015\ \text{mm}$.