The 37th International Physics Olympiad
Singapore

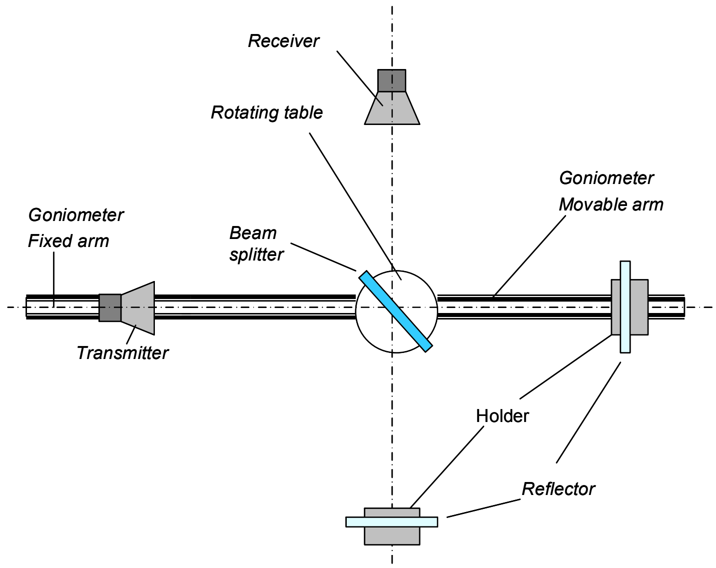
Experimental Competition

Wednesday, 12 July, 2006

Sample Solution

Part 1

a. A sketch of the experimental setup (not required)



b. Data sheet (not required)

Position (cm)	Meter reading (mA)	Position (cm)	Meter reading (mA)	Position (cm)	Meter reading (mA)	Position (cm)	Meter reading (mA)
104.0	0.609	100.9	1.016	96.0	0.514	91.0	0.925
103.9	0.817	100.85	1.060	95.8	0.098	90.9	1.094
103.8	0.933	100.8	1.090	95.6	0.192	90.8	1.245
103.7	1.016	100.7	0.994	95.4	0.669	90.7	1.291
103.6	1.030	100.6	0.940	95.3	0.870	90.6	1.253
103.5	0.977	100.4	0.673	95.2	1.009	90.4	0.978
103.4	0.890	100.2	0.249	95.1	1.119	90.2	0.462
103.3	0.738	100.0	0.074	95.0	1.138	90.0	0.045
103.2	0.548	99.8	0.457	94.9	1.080	89.8	0.278
103.1	0.310	99.6	0.883	94.7	0.781	89.6	0.809
103.0	0.145	99.4	1.095	94.5	0.403	89.5	1.031
102.9	0.076	99.3	1.111	94.3	0.044	89.4	1.235
102.8	0.179	99.2	1.022	94.1	0.364	89.3	1.277
102.7	0.392	99.0	0.787	93.9	0.860	89.2	1.298
102.6	0.623	98.8	0.359	93.7	1.103	89.1	1.252
102.5	0.786	98.6	0.079	93.6	1.160	89.0	1.133
102.4	0.918	98.4	0.414	93.5	1.159	88.8	0.684
102.3	0.988	98.2	0.864	93.4	1.083	88.6	0.123
102.2	1.026	98..0	1.128	93.2	0.753	88.5	-0.020
102.1	1.006	97.9	1.183	93.0	0.331	88.4	0.123
102.0	0.945	97.8	1.132	92.8	0.073	88.2	0.679
101.9	0.747	97.7	1.015	92.6	0.515	88.0	1.116
101.8	0.597	97.5	0.713	92.4	0.968	87.9	1.265
101.7	0.363	97.2	0.090	92.2	1.217	87.8	1.339
101.6	0.161	97.0	0.342	92.15	1.234	87.7	1.313
101.5	0.055	96.8	0.714	92.1	1.230	87.6	1.190
101.4	0.139	96.6	1.007	92.0	1.165	87.4	0.867
101.3	0.357	96.5	1.087	91.8	0.871	87.2	0.316

101.2	0.589	96.4	1.070	91.6	0.353	87.1	0.034
101.1	0.781	96.3	1.018	91.4	0.018	87.0	-0.018
101.0	0.954	96.2	0.865	91.2	0.394	86.9	0.178
104.0	0.609	100.9	1.016	96.0	0.514	91.0	0.925
103.9	0.817	100.8	1.060	95.8	0.098	90.9	1.094
103.8	0.933	100.8	1.090	95.6	0.192	90.8	1.245
103.7	1.016	100.7	0.994	95.4	0.669	90.7	1.291
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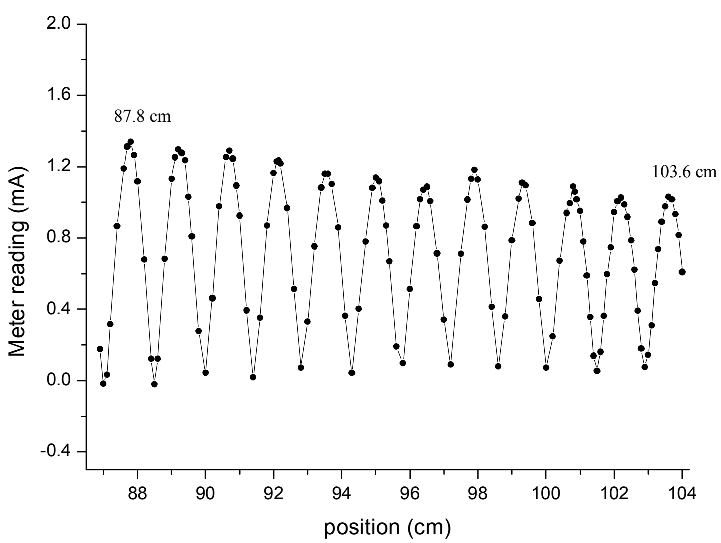
103.5	0.977	100.4	0.673	95.2	1.009	90.4	0.978
103.4	0.890	100.2	0.249	95.1	1.119	90.2	0.462
103.3	0.738	100.0	0.074	95.0	1.138	90.0	0.045
103.2	0.548	99.8	0.457	94.9	1.080	89.8	0.278
103.1	0.310	99.6	0.883	94.7	0.781	89.6	0.809
103.0	0.145	99.4	1.095	94.5	0.403	89.5	1.031
102.9	0.076	99.3	1.111	94.3	0.044	89.4	1.235
102.8	0.179	99.2	1.022	94.1	0.364	89.3	1.277
102.7	0.392	99.0	0.787	93.9	0.860	89.2	1.298
102.6	0.623	98.8	0.359	93.7	1.103	89.1	1.252
102.5	0.786	98.6	0.079	93.6	1.160	89.0	1.133
102.4	0.918	98.4	0.414	93.5	1.159	88.8	0.684
102.3	0.988	98.2	0.864	93.4	1.083	88.6	0.123
102.2	1.026	98.0	1.128	93.2	0.753	88.5	-0.020
102.1	1.006	97.9	1.183	93.0	0.331	88.4	0.123
102.0	0.945	97.8	1.132	92.8	0.073	88.2	0.679
101.9	0.747	97.7	1.015	92.6	0.515	88.0	1.116
101.8	0.597	97.5	0.713	92.4	0.968	87.9	1.265
101.7	0.363	97.2	0.090	92.2	1.217	87.8	1.339
101.6	0.161	97.0	0.342	92.15	1.234	87.7	1.313
101.5	0.055	96.8	0.714	92.1	1.230	87.6	1.190
101.4	0.139	96.6	1.007	92.0	1.165	87.4	0.867
101.3	0.357	96.5	1.087	91.8	0.871	87.2	0.316
101.2	0.589	96.4	1.070	91.6	0.353	87.1	0.034
101.1	0.781	96.3	1.018	91.4	0.018	87.0	-0.018
101.0	0.954	96.2	0.865	91.2	0.394	86.9	0.178



From the graph (not required) or otherwise, the positions of the first maximum point and 12^{th} maximum point are measured at 87.8 cm and 103.6 cm.

The wavelength is calculated by

$$\frac{\lambda}{2} = \frac{103.6 - 87.8}{11} \text{cm}$$

1.8 marks

Thus, $\boxed{\lambda = 2.87 \text{ cm}}$.

Error analysis

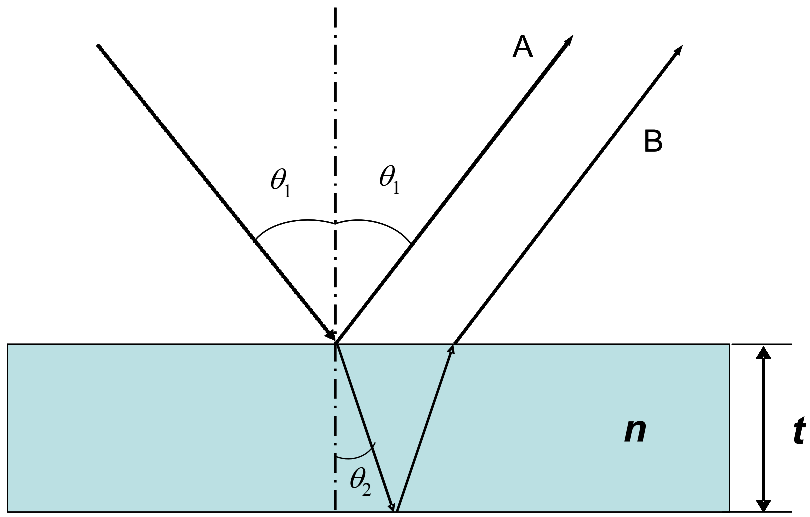
$$\lambda = \frac{2}{11} d \,, \qquad \Delta d = 0.05 \text{ x2 cm} = 0.1 \text{ cm.}$$

$$|\Delta\lambda| = \left|\frac{2}{11}\Delta d\right| = \frac{2}{11} \times 0.10 = 0.018 \quad cm < 0.02 \quad cm$$

0.2 marks

Part 2

(a) Deduction of interference conditions



Assume that the thickness of the film is t and refractive index n. Let θ_1 be the incident angle and θ_2 the refracted angle. The difference of the optical paths ΔL is:

$$\Delta L = 2(nt/\cos\theta_2 - t\tan\theta_2 \sin\theta_1)$$

Law of refraction:

$$\sin\theta_1 = n\sin\theta_2$$

Thus

$$\Delta L = 2t\sqrt{n^2 - \sin^2\theta_1}$$

Considering the <u>180 deg (π)</u> phase shift at the air- thin film interface for the reflected beam, we have interference conditions:

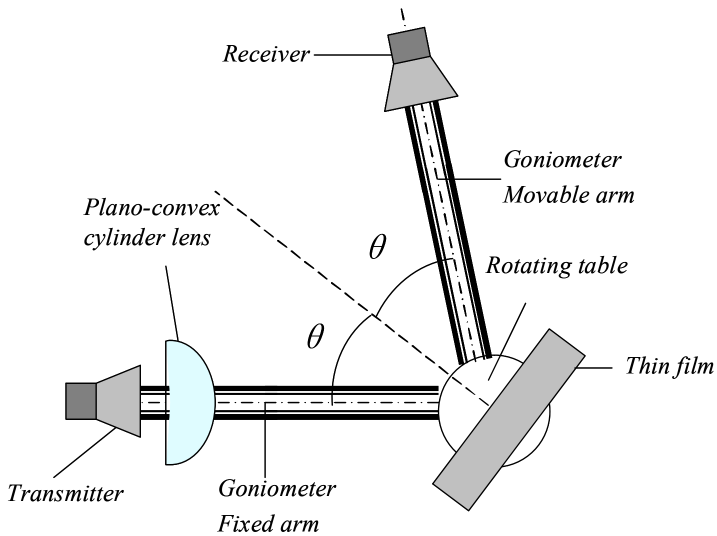
$$2t\sqrt{n^2 - \sin^2\theta_{\text{min}}} = m\lambda \quad (m = 1,2,3,...) \qquad \text{for the destructive peak}$$

and $$2t\sqrt{n^2 - \sin^2\theta_{\text{max}}} = (m \pm \frac{1}{2})\lambda \qquad \text{for the constructive peak}$$

1 mark

If thickness t and wave length λ are known, one can determine the refractive index of the thin film from I - θ_1 spectrum (I is the intensity of the interfered beam).

(b) A sketch of the experimental setup



1 mark

Students should use the labeling on Page 2.

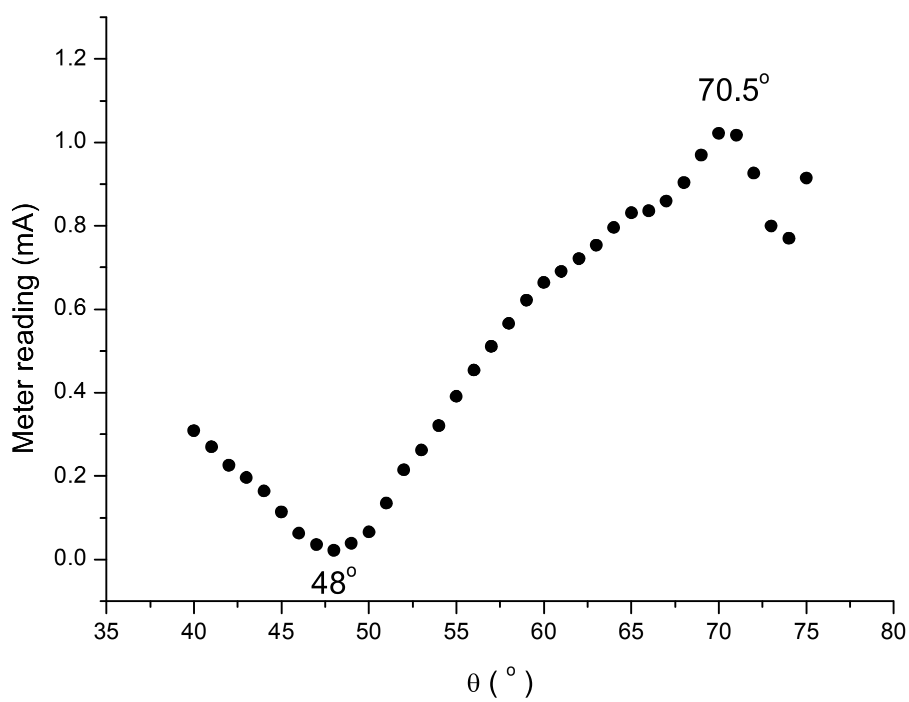
(c) Data Set

X: θ_I / degree	Y: Meter reading S/mA
40.0	0.309
41.0	0.270
42.0	0.226
43.0	0.196
44.0	0.164
45.0	0.114
46.0	0.063
47.0	0.036
48.0	0.022
49.0	0.039
50.0	0.066
51.0	0.135
52.0	0.215
53.0	0.262
54.0	0.321
55.0	0.391
56.0	0.454
57.0	0.511

58.0	0.566
59.0	0.622
60.0	0.664
61.0	0.691
62.0	0.722
63.0	0.754
64.0	0.796
65.0	0.831
66.0	0.836
67.0	0.860
68.0	0.904
69.0	0.970
70.0	1.022
71.0	1.018
72.0	0.926
73.0	0.800
74.0	0.770
75.0	0.915

0.5 marks

Uncertainty: angle $\Delta\theta_1 = \pm 0.5^o$, current: ±0.001 mA



From the data, θ_{min} and θ_{max} can be found at 48° and 70.5° respectively.

0.9 marks

To calculate the refractive index, the following equations are used:

0.6 marks

$$2t\sqrt{n^2 - \sin^2 48^o} = m\lambda \quad (m = 1,2,3,...) \qquad (1)$$

and

$$2t\sqrt{n^2 - \sin^2 70.5^o} = (m - \frac{1}{2})\lambda \qquad (2)$$

In this experiment, t = 5.28 cm, λ = 2.85cm (measured using other method).

Solving the simultaneous equations (1) and (2), we get

$$m = \frac{\sin^2 70.5^o - \sin^2 48^o}{(\frac{\lambda}{2t})^2} + 0.25$$

$$m = 4.83 \longrightarrow m = 5$$

1 mark

Substituting m = 5 in (1), we get n = 1.54

Substituting m = 5 in (2), we also get n = 1.54

0.5 marks

Error analysis:

$$n = \sqrt{\sin^2\theta + (\frac{m\lambda}{2t})^2}$$

$$\Delta n = \frac{1}{\sqrt{\sin^2\theta + (\frac{m\lambda}{2t})^2}}(\sin 2\theta \bullet \Delta\theta + \frac{m^2\lambda}{2t^2}\Delta\lambda - \frac{m^2\lambda^2}{2t^3}\Delta t)$$

$$= \frac{1}{n}(\sin 2\theta \bullet \Delta\theta + \frac{m^2\lambda}{2t^2}\Delta\lambda - \frac{m^2\lambda^2}{2t^3}\Delta t)$$

If we take $\Delta\theta = \pm 0.5^o = \pm 0.0087$ rad, $\Delta t = \pm 0.05$ cm, $\Delta\lambda = \pm 0.02$ cm, and $\theta = 48^o$

$$\Delta n = \frac{1}{1.54}(0.0087 \sin 96^o + \frac{5^2 \times 2.85}{2 \times 5.28^2} \times 0.01 + \frac{5^2 \times 2.85^2}{2 \times 5.28^3} \times 0.05) \approx 0.02$$

Thus, $\boxed{n + \Delta n = 1.54 \pm 0.02}$

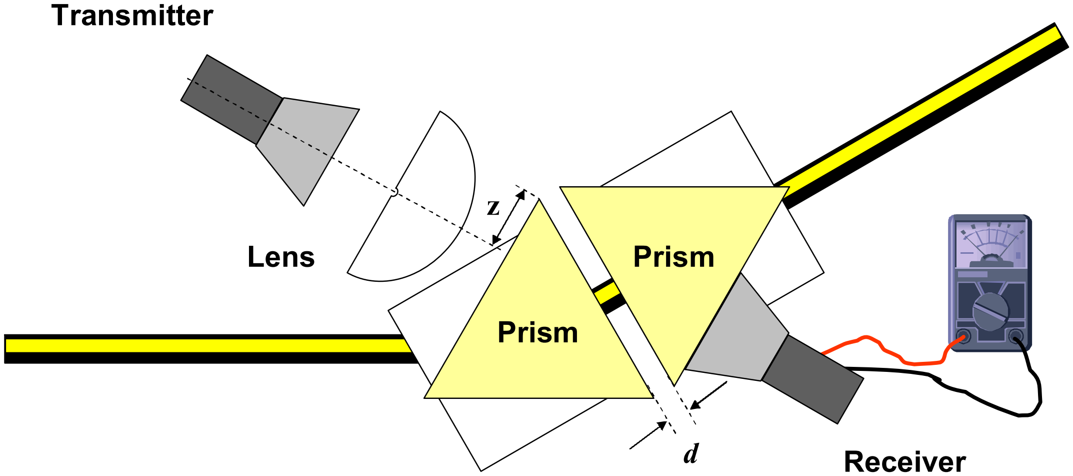
0.5 marks

Part 3

Sample Solution

Task 1

Sketch your final experimental setup and mark all components using the labels given at page 2. In your sketch, write down the distance z (see Figure 3.2), where z is the distance from the tip of the prism to the central axis of the transmitter.



(Students should use labels on page 2.)

Task 2

Tabulate your data. Perform the experiment twice.

Data Set

X: d(cm)	ΔX(cm)	*Set 1* S_1 (mA)	*Set 2* S_2(mA)	$S_{average}$ (mA)	ΔS(mA)#	I_t (mA)^2*	$\Delta(I_t)$$	Y: ln(I_t (mA)^2)	ΔY&
0.60	0.05	0.78	0.78	0.780	0.01	0.6080	0.016	-0.50	0.03
0.70	0.05	0.68	0.69	0.685	0.01	0.4690	0.014	-0.76	0.03
0.80	0.05	0.58	0.59	0.585	0.01	0.3420	0.012	-1.07	0.03
0.90	0.05	0.50	0.51	0.505	0.01	0.2550	0.010	-1.37	0.04
1.00	0.05	0.42	0.42	0.420	0.01	0.1760	0.008	-1.74	0.05
1.10	0.05	0.36	0.35	0.355	0.01	0.1260	0.007	-2.07	0.06
1.20	0.05	0.31	0.31	0.310	0.01	0.0961	0.006	-2.34	0.06
1.30	0.05	0.26	0.25	0.255	0.01	0.0650	0.005	-2.73	0.08
1.40	0.05	0.21	0.22	0.215	0.01	0.0462	0.004	-3.07	0.09

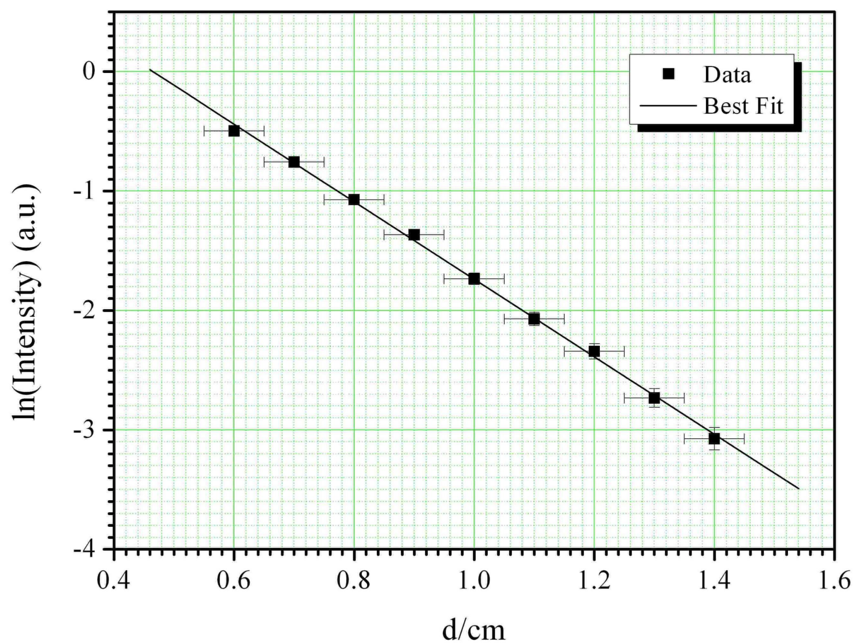
$\Delta S = 0.01$ mA (for each set of current measurements)

* S^2 proportional to the intensity, I_t

$ $\Delta(S^2) = \Delta I_t = 2\, S \times \Delta S$

& $\Delta Y = \Delta(\ln I_t) = \Delta(I_t)/I_t$

Task 3
By plotting appropriate graphs, determine the refractive index, n_1, of the prism with error analysis. Write the refractive index n_1, and its uncertainty Δn_1, of the prism in the answer sheet provided.



Least Square Fitting

X = *d*(cm)	ΔX(cm)	Y = ln(I_t)	ΔY	ΔY^2	XY	X^2	Y^2
0.60	0.05	-0.50	0.03	0.001	-0.298	0.360	0.247
0.70	0.05	-0.76	0.03	0.001	-0.530	0.490	0.573
0.80	0.05	-1.07	0.03	0.001	-0.858	0.640	1.150
0.90	0.05	-1.37	0.04	0.002	-1.230	0.810	1.867
1.00	0.05	-1.74	0.05	0.002	-1.735	1.000	3.010
1.10	0.05	-2.07	0.06	0.003	-2.278	1.210	4.290
1.20	0.05	-2.34	0.06	0.004	-2.811	1.440	5.487
1.30	0.05	-2.73	0.08	0.006	-3.553	1.690	7.469
1.40	0.05	-3.07	0.09	0.009	-4.304	1.960	9.451
ΣX =		ΣY =	ΣΔY =	$\Sigma(\Delta Y)^2$ =	ΣXY =	ΣX^2 =	ΣY^2 =
9.00		-15.648	0.469	0.029	-17.596	9.600	33.544

From $I_t = I_0 \exp(-2\gamma d)$, taking natural log on both sides, we obtain:

$$\ln(I_t) = -2\gamma d + \ln(I_0)$$

which is of the form $y = mx + c$.

To calculate the gradient, the following equation was used, where N is the number of data points:

$$m = \frac{N\sum(XY) - (\sum X)(\sum Y)}{N\sum X^2 - (\sum X)^2} = -3.247$$

To calculate the standard deviation σ_Y of the individual Y data values, the following equation was used:

$$\sigma_Y = \sqrt{\frac{\sum(\Delta Y)^2}{N-2}} = 0.064$$

Hence the standard deviation in the slope can be calculated:

$$\sigma_m = \sigma_Y \sqrt{\frac{N}{N\sum X^2 - (\sum X)^2}} = 0.082$$

From the gradient:

$$\begin{aligned} 2\gamma &= 3.247 \pm 0.082 \\ &\approx 3.25 \pm 0.08 \end{aligned}$$

Using:

$$n_1 = \frac{\sqrt{k_2^2 + \gamma^2}}{k_2 \sin\theta_1}$$

where $\theta_1 = 60^{\circ}$, $k_2 = 2\pi/\lambda \approx 2.20$ (using the wavelength determined from earlier part (using $\lambda = (2.85 \pm 0.02)$cm), we obtain:

$$\boxed{\begin{aligned} n_1 \pm \Delta n_1 &= 1.434 \pm 0.016 \\ &\approx 1.43 \pm 0.02 \end{aligned}}$$

Error Analysis for refractive index of n_1

$$\Delta n_1 = \frac{d}{dk_2}\left[\frac{\left(k_2^2+\gamma^2\right)^{1/2}}{k_2 \sin\theta_1}\right]\Delta k_2 + \frac{d}{d\gamma}\left[\frac{\left(k_2^2+\gamma^2\right)^{1/2}}{k_2 \sin\theta_1}\right]\Delta\gamma$$

$$\begin{aligned}\Delta n_1 &= \left[\frac{\left(k_2^2+\gamma^2\right)^{-1/2}}{\sin\theta_1} - \frac{\left(k_2^2+\gamma^2\right)^{1/2}}{k_2^2 \sin\theta_1}\right]\Delta k_2 + \left[\frac{\gamma\left(k_2^2+\gamma^2\right)^{-1/2}}{k_2 \sin\theta_1}\right]\Delta\gamma \\ &= 0.016 \\ &\approx 0.02\end{aligned}$$

where:

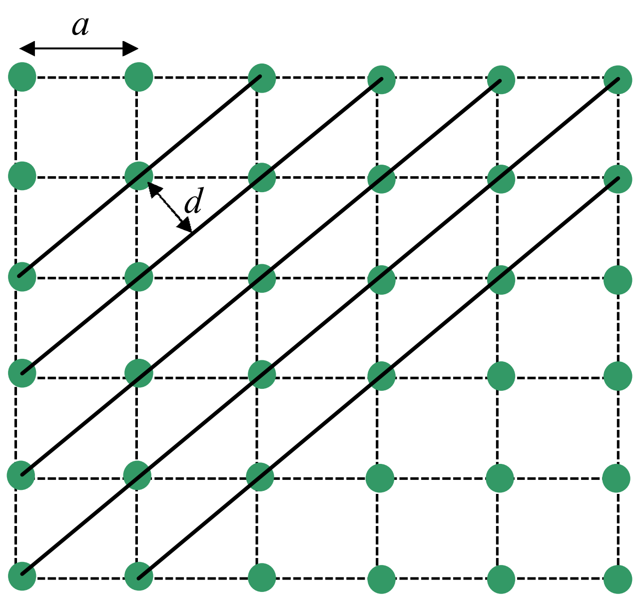
$$\Delta k_2 = -\frac{2\pi}{\lambda^2}\Delta\lambda = -0.015$$

Note: Other methods of error analysis are also accepted.

Part 4

Task 1

Top-view of a simple square lattice.



0.5 marks

Figure 4.1: *Schematic diagram of a simple square lattice with lattice constant a and interplaner d of the diagonal planes indicated.*

Deriving Bragg's Law

Conditions necessary for the observation of diffraction peaks:

1. The angle of incidence = angle of scattering.
2. The pathlength difference is equal to an integer number of wavelengths.

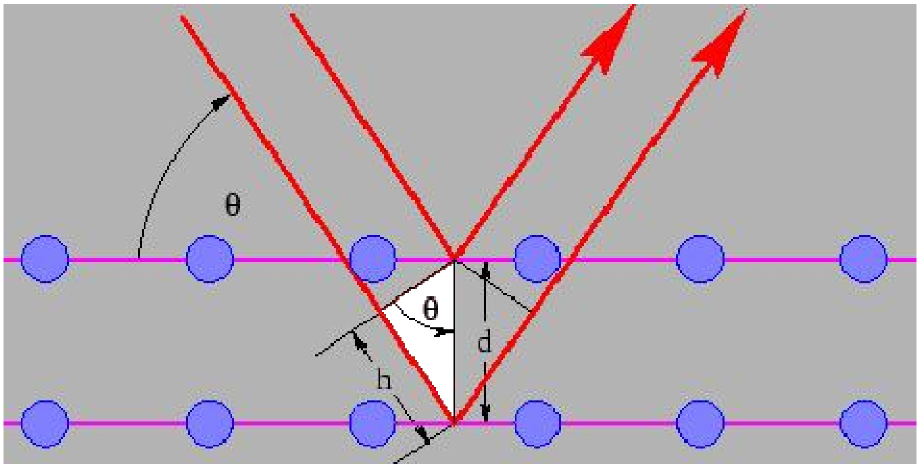


Figure 4.2: *Schematic diagram for deriving Bragg's law.*

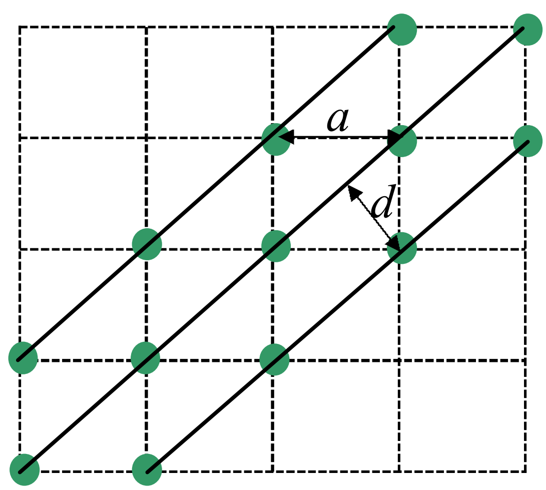
$h = d \sin\theta$ (1).

The path length difference is given by,

$2h = 2d \sin\theta$ (2).

For diffraction to occur, the path difference must satisfy,

$\boxed{2\, d \sin\theta = m\lambda,}$ $m = 1, 2, 3...$ (3).



0.5 marks

Figure 4.3 *Illustration of the lattice used in the experiment (this Figure is not required)*

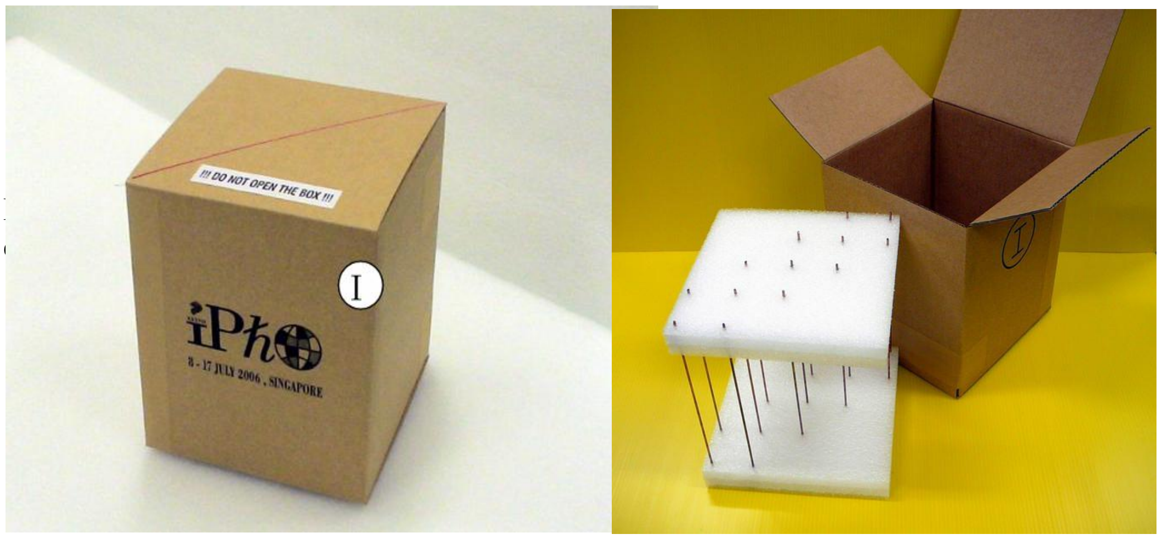
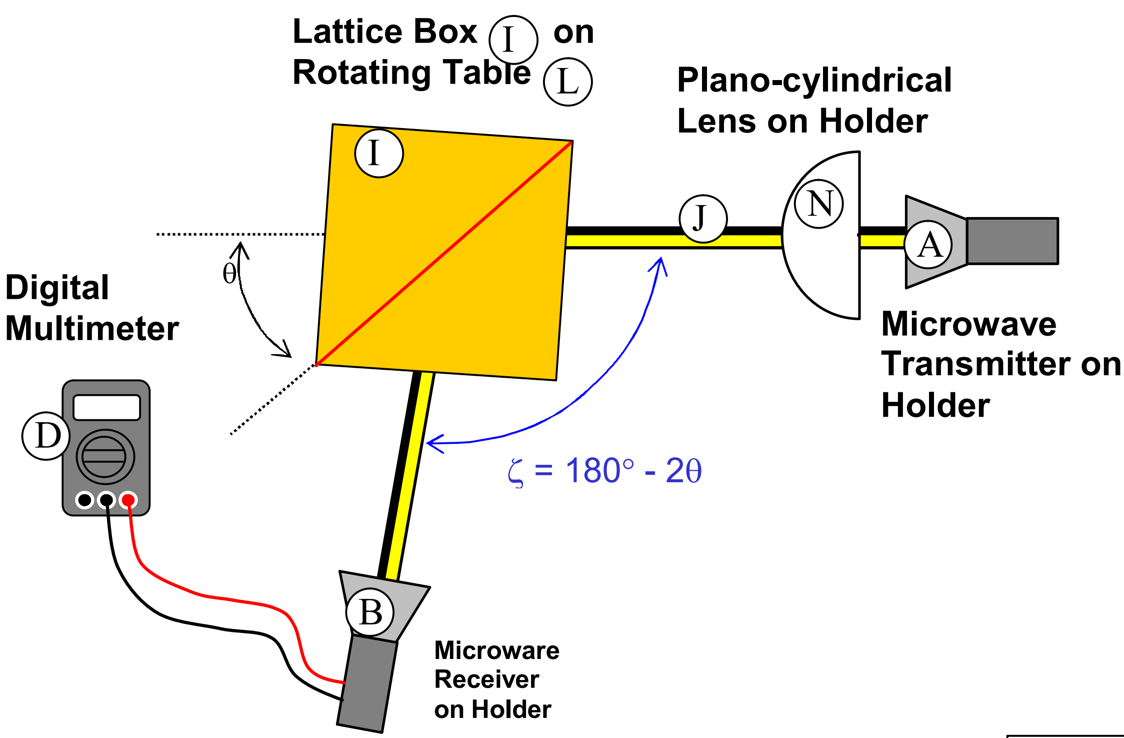


__Fig. 4.4__ The actual lattice used in the experiment (not required)

Task 2 (a)



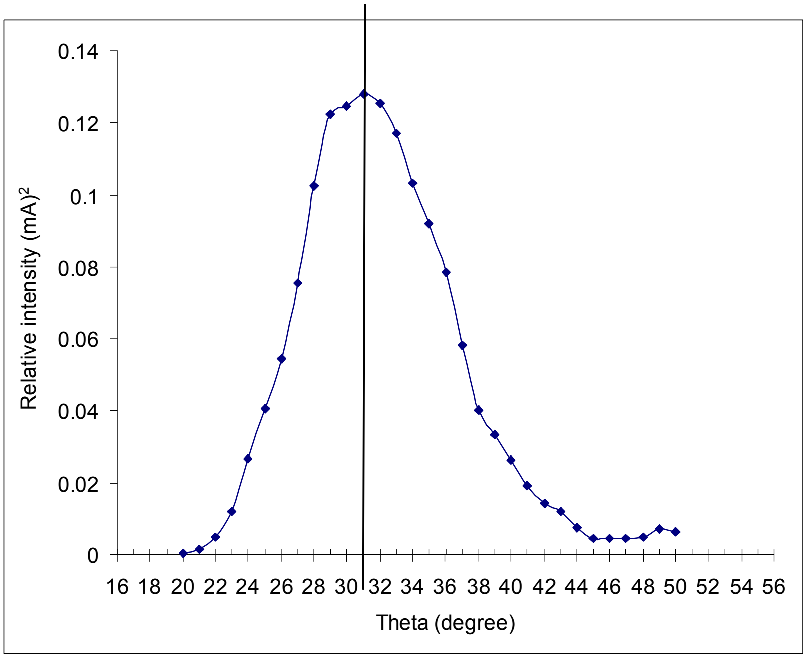
1.5 marks

Fig. 4.5 Sketch of the experimental set up

Task 2(b) & 2(c)

Data Set

θ/°	ζ/°	Output current S (mA)	Intensity $I=S^2$ $(mA)^2$
20.0	140.0	0.023	0.000529
21.0	138.0	0.038	0.001444
22.0	136.0	0.070	0.0049
23.0	134.0	0.109	0.011881
24.0	132.0	0.163	0.026569
25.0	130.0	0.201	0.040401
26.0	128.0	0.233	0.054289
27.0	126.0	0.275	0.075625
28.0	124.0	0.320	0.1024
29.0	122.0	0.350	0.1225
30.0	120.0	0.353	0.124609
31.0	118.0	0.358	0.128164
32.0	116.0	0.354	0.125316
33.0	114.0	0.342	0.116964
34.0	112.0	0.321	0.103041
35.0	110.0	0.303	0.091809
36.0	108.0	0.280	0.0784
37.0	106.0	0.241	0.058081
38.0	104.0	0.200	0.04
39.0	102.0	0.183	0.033489
40.0	100.0	0.162	0.026244
41.0	98.0	0.139	0.019321
42.0	96.0	0.120	0.0144
43.0	94.0	0.109	0.011881
44.0	92.0	0.086	0.007396
45.0	90.0	0.066	0.004356
46.0	88.0	0.067	0.004489
47.0	86.0	0.066	0.004356
48.0	84.0	0.070	0.0049
49.0	82.0	0.084	0.007056
50.0	80.0	0.080	0.0064



2.7 marks

Task 2(d)

From eq 3 and let $m = 1$,

$$2d \sin \theta_{max} = \lambda \qquad (4)$$

From Fig. 4.3,

$a = \sqrt{2}d$ (5)

Combine eqs (4) and (5), we obtain,

$$\boxed{a = \frac{\lambda}{\sqrt{2}\sin\theta_{\max}}}$$

From the symmetry of the data, the peak position is determined to be:

$\theta_{max} = 31^{o}$ (The theoretical value is $\theta_{max} = 32^{o}$)

$$\boxed{a = \frac{\lambda}{\sqrt{2}\sin\theta_{\max}} = \frac{2.85cm}{\sqrt{2}\sin 31^{o}} = 3.913cm}$$

(<u>Actual value a = 3.80 cm</u>)

[*The value 3.55 in the marking scheme is derived from:*

$$a = \frac{\lambda}{\sqrt{2}\sin\theta_{\max}} = \frac{2.83cm}{\sqrt{2}\sin 34^{o}} = 3.58cm$$

where 2.83 cm and 34 deg are the min and max allowed values for wavelength and peak position.

Similarly:

The value 4.10 is derived from: $a = \frac{\lambda}{\sqrt{2}\sin\theta_{\max}} = \frac{2.87cm}{\sqrt{2}\sin 30^{o}} = 4.06cm$

The value 3.55 is derived from: $a = \frac{\lambda}{\sqrt{2}\sin\theta_{\max}} = \frac{2.83cm}{\sqrt{2}\sin 34^{o}} = 3.58cm$

The value 3.40 is derived from: $a = \frac{\lambda}{\sqrt{2}\sin\theta_{\max}} = \frac{2.83cm}{\sqrt{2}\sin 35^{o}} = 3.49cm$

The value 4.20 is derived from: $a = \frac{\lambda}{\sqrt{2}\sin\theta_{\max}} = \frac{2.87cm}{\sqrt{2}\sin 29^{o}} = 4.18cm$]

Error analysis:

Known uncertainties:
$\Delta\lambda = 0.02$ cm;

$\Delta\theta = 0.5$ deg = 0.014 rad. (uncertainty in determining θ from graph).

From: $a = \dfrac{\lambda}{\sqrt{2}\sin\theta_{\max}}$

$$\Delta a = \frac{\Delta\lambda}{\sqrt{2}\sin\theta_{\max}} - \frac{\lambda}{\sqrt{2}(\sin\theta_{\max})^2}\frac{d}{d\theta}(\sin\theta_{\max})\Delta\theta$$

$$= a(\frac{\Delta\lambda}{\lambda} - \frac{1}{\sin\theta_{\max}}\frac{d}{d\theta}(\sin\theta_{\max})\Delta\theta)$$

$$= a(\frac{\Delta\lambda}{\lambda} - \cot\theta_{\max}\Delta\theta)$$

$$= 3.80(\frac{0.02}{2.85} - \cot(32°)\times(-0.014))cm$$

$$= 0.112cm \approx 0.1$$

Hence:

$$a \pm \Delta a = 3.913 \pm 0.112$$
$$\approx 3.9 \pm 0.1 \text{ cm}$$

0.8 marks