MARK SCHEME AND SOLUTIONS FOR Q3

Total marks = 10

A **a)** $\Delta x_t = ae^{-\mu t} \cos(\omega t + \phi)$, $0.8 = e^{-50\mu} \Rightarrow \mu = 4.5 \times 10^{-3}$ s^{-1}. [0.1]

b) $\upsilon = (E/\rho)^{1/2} = (7.1 \times 10^{10}/2700)^{1/2} = 5100$ m.s^{-1}.
At fundamental $\lambda_{rod} = 4l = 4$ m.
$f = 5100 / 4 = 1.3 \times 10^3$ Hz.
$\omega = 2\pi f = 8.1 \times 10^3$ rad.s^{-1}. [0.1]

c) $\upsilon = f\lambda_{rod}$, $\delta\lambda_{rod} / \lambda_{rod} = (-)\delta f / f \Rightarrow \delta l / l$. [0.8]
$\delta l = l.(\delta f / f)$. [0.6]
$\delta l = 1 \times (5.0 \times 10^{-3}/1.3 \times 10^3) = 3.8 \times 10^{-6}$ m. [0.1]

d) Change in gravitational force on rod at a distance x from the free end = $m\Delta g$ and $m = \rho x A$,
where A is the cross-sectional area of the rod. [0.5]
Change in stress = $m\Delta g/A = \rho x \Delta g$. [0.5]
Change in strain = $\delta(dx)/dx = \rho x \Delta g/E$;
that is, $dx \rightarrow (1 + \rho x \Delta g/E)dx \Rightarrow \Delta l = (\rho \Delta g/2E)l^2$. [0.5]

e) At fundamental $\lambda_{rod} = 4l \Rightarrow \Delta l = \Delta\lambda_{rod}/4$,
for $\Delta\lambda_{rod} = 656$ nm$/10^4 \Rightarrow \Delta l = 656$ nm $/(4 \times 10^4)$. [0.1]
$\Delta l = 656$ nm $/(4 \times 10^4) = (\rho\Delta g/2E)l^2$ [0.1]
$\Delta l = (2700 \times 10^{-19}/ 14 \times 10^{10})\, l^2 \Rightarrow l = 9.2 \times 10^7$ m. [0.1]

B **a)** $mc^2 = hf \Rightarrow m = hf/c^2$, [0.3]
$hf' = hf - GMm/R$, [0.3]
$\Rightarrow hf' = hf(1 - GM/Rc^2)$, $\therefore f' = f(1 - GM/Rc^2)$. [0.4]

b) $n_r = c / c(1 - GM/rc^2)^2$, [1.0]
$n_r = 1 + 2GM/rc^2$, for small GM/rc^2; i.e. $\alpha = 2$. [1.0]

c)

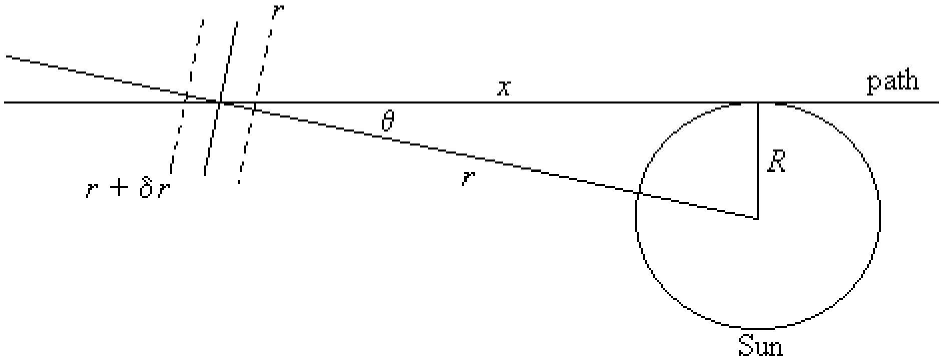

Diagram [0.2]

By Snell's law: $n(r + \delta r) \sin\theta = n(r) \sin(\theta - \delta\xi)$, [1.0]

$(n(r) + (dn/dr)\,\delta r) \sin\theta = n(r) \sin\theta - n(r) \cos\theta\, \delta\xi$. [0.4]

$(dn/dr)\,\delta r \sin\theta = -\, n(r) \cos\theta\, \delta\xi$.

Now $n(r) = 1 + 2GM/rc^2$, so $(dn/dr) = -\,2GM/c^2r^2$, [0.3]

and $(2GM/c^2r^2) \sin\theta\, \delta r = n(r) \cos\theta\, \delta\xi$.

Hence $\delta\xi = (2GM/c^2r^2) \tan\theta\, (\delta r/n) \approx (2GM \tan\theta /c^2r^2)\delta r$. [1.0]

Now $r^2 = x^2 + R^2$, so $rdr = xdx$. [0.1]

$$\int d\xi = \frac{2GM}{c^2}\int\frac{\tan\theta dr}{r^2} = \frac{2GM}{c^2}\int\frac{\tan\theta r dr}{r^3} = \frac{2GMR}{c^2}\int_{-\infty}^{+\infty}\frac{dx}{(x^2+R^2)^{3/2}}$$

$$\xi = \frac{4GM}{Rc^2}\,radians = 8.4\times10^{-6}\,radians.$$

[0.5]