APhO 2024 – Theory Question No.2:

A1.

The velocity of light propagating inside the spheres is shown below:

$$u_x = \frac{c}{n_x}$$

With c as the speed of light and u_x the propagating speed in the semi-sphere.

Because of Snell's law: $n_x \sin\theta_i = n_2 \sin\theta_x$ and because $\theta_a < \theta_b$, we can easily conclude:

$$n_a < n_b$$

From there, we have:

$$u_a > u_b$$

This means that colour A propagates faster in the semi-sphere.

A2.

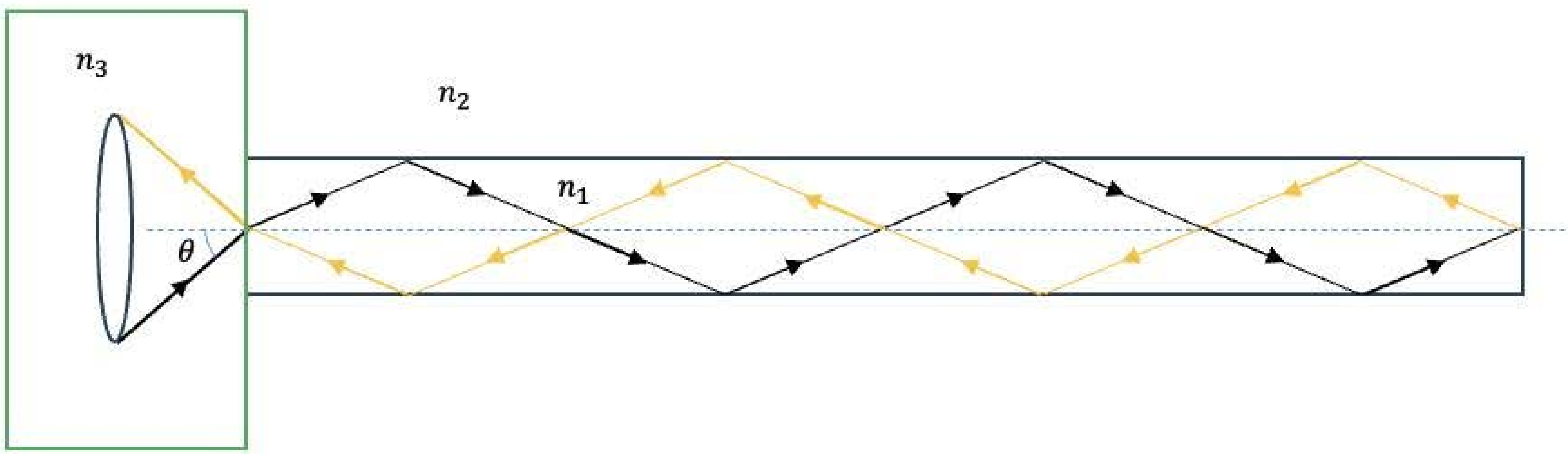
When the rays don't appear on the bottom of the semi-sphere, it means that θ_i has reached the critical angle.

The limit to the incident angle: $\sin\theta_i \leq \frac{1}{n_x}$

From this definition, we have: $n_b = \csc\frac{\pi}{4} = \sqrt{2} = 1{,}412$ and $n_a = \csc\frac{5\pi}{18} = 1{,}305$

So, $n_b - n_a = 0{,}107$

B1.



The critical angle of the rod-air surface:

$$\theta_{12} = \arcsin\frac{n_2}{n_1} = 0{,}73$$

Snell's Law for the rod – polymer surface in the upper limit (Reflect through the sides of the polymer)

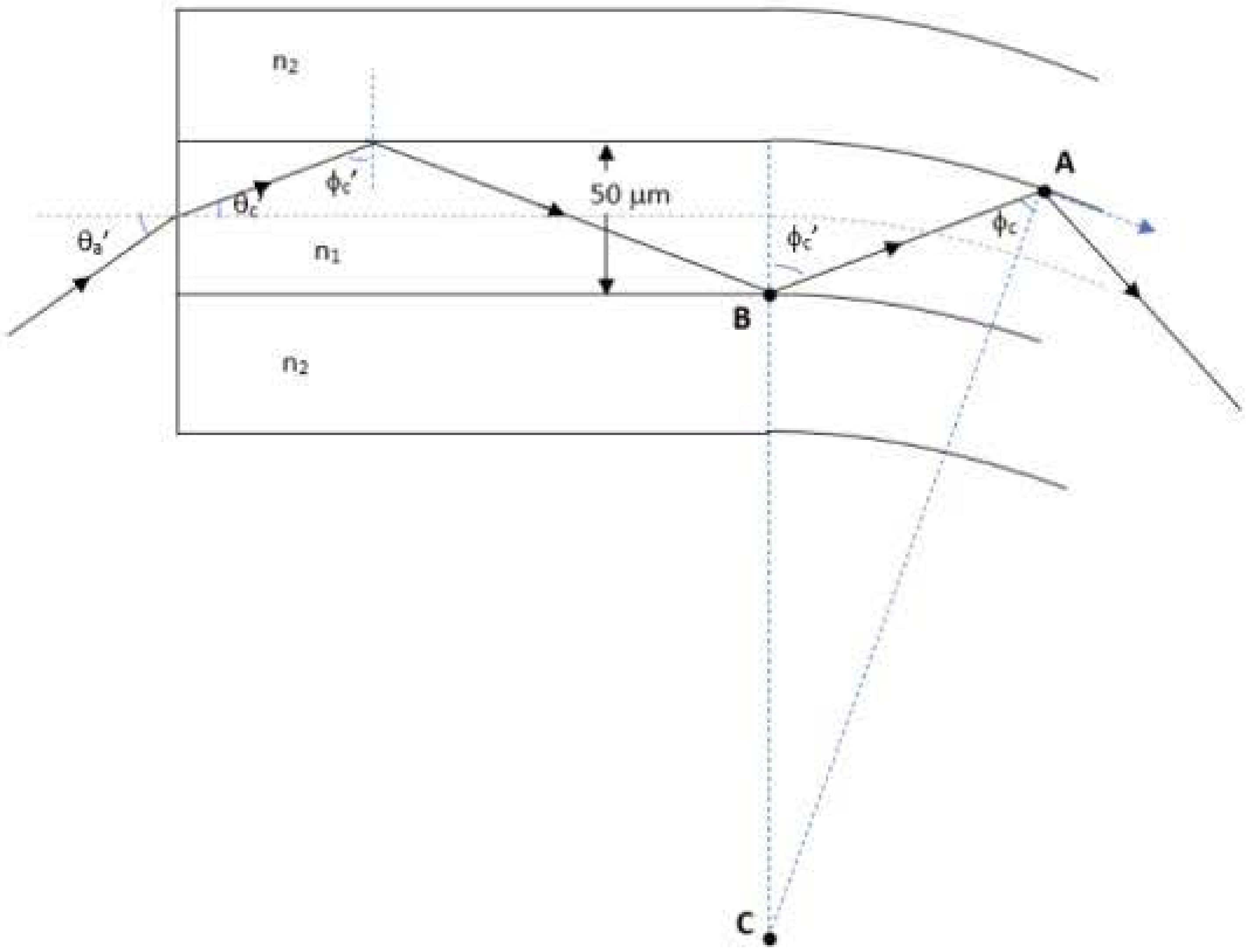
$$n_3 \sin\theta_a = n_1 \sin\theta_b$$

$$n_1 \sin(\frac{\pi}{2} - \theta_b) = n_2 \sin\theta_c = n_2$$

The maximum incident angle θ_a:

$$\theta_a = \arcsin(\frac{n_1}{n_3}\sqrt{1 - \frac{{n_2}^2}{n_1}})$$

C2.



Because the rod is curved with radius R and inner diameter d so we have:

$$\frac{\sin\phi_c}{R - \frac{d}{2}} = \frac{\sin{\phi_c}'}{R + \frac{d}{2}}$$

$$\sin{\phi_c}' = \sin\phi_c \frac{2R + d}{2R - d}$$

Also:

$$n_3 \sin{\theta_a}' = n_1 \sin\theta_c$$

$$\theta_c = \frac{\pi}{2} - {\phi_c}'$$

$$n_1 \sin\phi_c = n_2$$

From the equations above, we have the maximum incident angle ${\theta_a}'$:

$$n_3 \sin\theta = n_1 \sin\left(\frac{\pi}{2} - \theta_{12}\right) = n_1 \cos\theta_{12}$$

The upper limit:

$$\theta \le \arcsin\left(\frac{n_1}{n_3}\cos\theta_{12}\right) = 0{,}92$$

Snell's Law for the rod – polymer surface in the lower limit (Reflect through the other end of the polymer)

$$n_3 \sin\theta = n_1 \sin\theta_{12}$$

The lower limit:

$$\theta \ge \arcsin\left(\frac{n_1}{n_3}\sin\theta_{12}\right) = 0{,}796$$

The range of incident angle θ is from 0,796 to 0,92 radians.

B2.

i) The critical angle of the light rays passing through the oil – rod surface does not exist for rays passing from rod to oil ($n_{oil} > n_1$)
This shows that light cannot be totally reflected back to the polymer since it is always refracted.

ii) The critical angle of the rod-water surface:

$$\theta_{1-water} = \arcsin\frac{n_{water}}{n_1} = 1{,}09$$

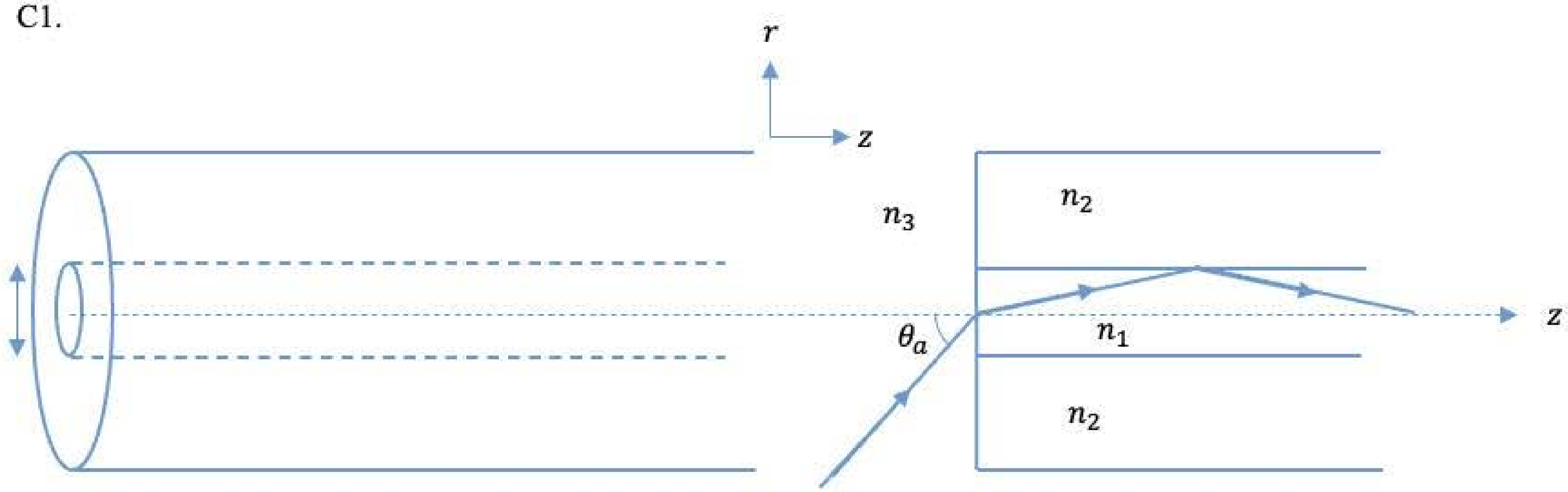
The limits of the two conditions in B1:

$$\theta \ge \arcsin\left(\frac{n_1}{n_3}\sin\theta_{1-water}\right) = 1{,}25$$

$$\theta \le \arcsin\left(\frac{n_1}{n_3}\cos\theta_{1-water}\right) = 0{,}518$$

The two equations above show that light cannot be totally reflected back to the polymer.

C1.



Snell's Law for rays passing through the air - optical fibre and optical fibre core – cladding so that no light leaves the core: